## Permutations

## Name

$\square$ Class

## Question 1

(a) At a school careers evening, five speakers - Alice, Brian, Ciara, David and Erica - have been invited to speak to students about their own professions.

All five speakers will be invited to speak, one after the other.
(i) In how many different orders can they be invited to speak?
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(ii) How many different speaking orders are possible if David is too nervous to speak first?

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Alice has another speaking engagement later that same night and so has to leave early. She cannot speak last or second last.
(iii) How many different speaking orders are possible in this case?
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Erica and Ciara intend to speak about professions that are similar.
(iv) How many speaking orders are possible that would have Erica and Ciara speaking one immediately after the other? [5]

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(v) How many different speaking orders are possible in which the three women do not speak one after the other?

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(vi) How many different speaking orders are possible in which Brian speaks before Erica does?

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(b) A PIN number for a bank ATM consists of four digits using the digits: 1, 2, 3, 4, 5, 6, 7 and 8 . No digit may be repeated in any PIN number. 1500 customers apply for ATM cards and the bank wants to give each customer their own unique four digit PIN number.
(i) Will this be possible with this system?

Explain.
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(ii) How many of these PIN numbers will consist of odd digits only?
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(iii) How many PIN numbers will be even numbers?

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(iv) How many PIN numbers will be between 3000 and 5000?
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(v) How many PIN numbers will be odd and between 3000 and 5000? [10]

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Another 900 customers apply for ATM cards. The bank still wants to ensure that each customer has their own unique PIN number.
(vi) Suggest two ways in which the bank could change their ATM PIN numbers to accommodate the extra customers.
[10] Explain your answer in each case.

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## Question 2

(a) (i) How many four letter arrangements are possible using the letters of the word TEACH if no repetitions are allowed? [5]

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(ii) If all of these arrangements are listed alphabetically, which one will come first?

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(iii) On this alphabetical list, how arrangements will come before the word EACH?

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(b) (i) How many arrangements are possible using the letters of the word TEACHER?

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(ii) How many of these arrangements do not contain a double E (one E after another)?


