LC OL – Probability

A Ryan

Learning Outcomes

As part of Leaving Certificate Ordinary Level Math you should be able to complete the following.

Counting

- List outcomes of an experiment
- > Apply the fundamental principle of counting
- Count the arrangements of n distinct objects (n!)
- Count the number of ways of arranging r objects from n distinct objects

Concepts of probability

- Decide whether an everyday event is likely or unlikely to occur
- Recognise that probability is a measure on a scale of 0-1 of how likely an event is to occur
- Use set theory to discuss experiments, outcomes, sample spaces
- > Use the language of probability to discuss events, including those with equally likely outcomes
- Estimate probabilities from experimental data
- Recognise that, if an experiment is repeated, there will be different outcomes and that increasing the number of times an experiment is repeated generally leads to better estimates of probability
- > Associate the probability of an event with its long run relative frequency
- > Discuss basic rules of probability (AND/OR, mutually exclusive) through the use of Venn diagrams
- Calculate expected value and understand that this does not need to be one of the outcomes
- Recognise the role of expected value in decision making and explore the issue of fair games

Outcomes of Random Processes

- > Construct sample spaces for two independent events
- Apply the principle that in the case of equally likely outcomes the probability is given by the number of outcomes of interest divided by the total number of outcomes (examples using coins, dice, spinners, urns with coloured objects, playing cards, etc.)
- > Find the probability that two independent events both occur
- Apply an understanding of Bernoulli trials (A Bernoulli trial is an experiment whose outcome is random and can be either of two possibilities: "success" or "failure")
- Solve problems involving up to 3 Bernoulli trials
- > Calculate the probability that the 1st success occurs on the *nth* Bernoulli trial where *n* is specified

A member of one direction is chosen at random, what are the possible outcomes?

A coin is flipped two times. How many outcomes are possible?

Francis flips a coin and rolls a die. List all the possible outcomes. How many outcomes are possible?

Maggie is trying on a dress in River Island, which comes in five different sizes and seven different colours. How many different dresses can Maggie try on?

Rating Probabilities

Assign one of the following possibilities to the statements below.

- Impossible.
- Unlikely.
- Even chance (fifty-fifty).
- Likely.
- Certain.

	-	
1.	It will get dark tonight	
2.	If I roll a dice, I will get an odd number.	
3.	It will rain tomorrow.	
4.	The sun will rise next week.	
5.	I will win the lottery.	
6.	I will get a 'head' when I toss a coin.	
7.	A baby being born and being a boy.	
8.	Offaly winning the next all Ireland football title.	
9.	Manchester United winning the premier league.	
10.	You will watch television tonight.	
11.	You will eat some food today.	
12.	If I roll a dice I will get an even number.	
13.	Ireland will win the next soccer world cup.	
14.	Tomorrow will be 40 degrees.	
15.	Kildare will win an All-Ireland Hurling title in the next 4 years	
16.	Finding a triangle with 4 sides.	
17.	Tomorrow a baby will be born in Ireland.	
18.	The next winner of the Lotto will be a woman.	
19.	You score 8 when you roll an ordinary dice.	
20.	It will rain in Galway sometime in the next week .	
21.	Someone in your class will be absent one day next week.	
22.	The next baby born will be a boy.	
23.	Ireland will win the next soccer World Cup.	

Practical Activity 1

Probability Roll a dice

Total number of possible outcomes = _____

	Predicted res	ults		Actual resul	lts
	desired	probability	expected in	obtained in	30
	outcomes	producting	30 rolls	rolls	
P(1)					
P(less than 3)					
P(2 or 5)					
P(even)					
P(not 4)					
P(between 1 and 6)					
P(5)					
P(not 6)					
P(8)					

Comment on how actual results compare with predicted results.

Practical Activity 2

Working in groups of two and sharing the work carry out the following tasks and keep the appropriate records in each case.

1.	Toss a coin 12 times a	nd put the results in	the frequency table.
± .		na pat the results h	i the negacity table.

Toss No	1	2	3	4	5	6	7	8	9	10	11	12
Result												

How many 'Heads' out of 12? ____ How many 'Tails' out of 12 ____ If you tossed 200 times how many heads would you expect? _____

2. Throw a 1 to 6 (6 sided) dice 15 times and record the result in the frequency table.

Throw No	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Result															
How many times	s did ea	ach no	show	? 1_	2		3	4	5_	6					
If	d:	00 1:				، ما بد ام ا ،		h	- In	`					

If you threw the dice 300 times how often would the number 3 show? _____

3. Select the following cards from a pack. 5 of spades (5S), 10 of spades (10S), 6 of Diamonds (6D), Queen of Clubs (QC) and King of Clubs (KC) Shuffle the cards and face them down on the desk - pick one and record the result. Repeat the procedure 20 times and complete the frequency chart by putting an x in the appropriate space.

Try No	1	2	3	4	5	6	7	8	9	1	1	1	1	1	1	1	1	1	1	2
										0	1	2	3	4	5	6	7	8	9	0
5 of spades																				
6																				
diamond																				
10 spades																				
Q Club																				
K Clubs																				
																				11
How many tir	nes (did e	ach d	card	show	/?	5S		6D		10	S		QC		кс				
How many tir									-											
How many tir			•																	
-			-				• _													
How many tir	nes	uid a	dian	nono	sno	wr														

Worksheet 1

Elaine has decided that she will buy a top and a pair of trousers. She has a choice of a pick, yellow or green top and grey or blue trousers. List all the possible colours of clothes that she could wear.

The list of all possible outcomes would be:

List all the possible colours of clothes that she could wear using a two-way table.

A decorator can choose to paint the door of a room in two colours, the walls in five colours and the ceiling in three colours. How many possible ways can she can paint the room?

Worksheet 2

You roll a 6-sided die.



What is P(2)? Write your answer as a percentage.

P(2) = %

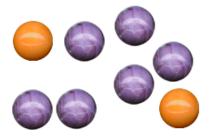
In a 15-sheet package of construction paper, there are 5 green sheets.

What is the probability that a randomly selected sheet will be green?

Simplify your answer and write it as a fraction or whole number.

P(green) =

You pick a marble at random.



What is P(orange)? Write your answer as a percentage.

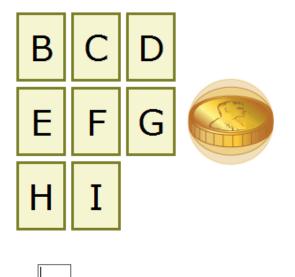
P(orange) = %

Of the last 12 contestants on a game show, 8 won a prize. What is the experimental probability that the next contestant will win a prize?

Simplify your answer and write it as a fraction or whole number.

P(win) =

You pick a card and flip a coin. How many outcomes are possible?



Ronice is building a snowman. There are 5 hats and 3 scarves to choose from. If Ronice uses one of each item, how many different snowmen can she make?

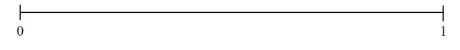
snowmen

A, B, C, D and E represent the probabilities of certain events occurring.

Event		Probability
A club is selected in a random draw from a pack of playing cards	А	
A tossed fair coin shows a tail on landing	В	
The sun will rise in the east tomorrow	С	
May will follow directly after June	D	
A randomly selected person was born on a Thursday	Е	

(a) Write the probability of each of the events listed into the table below.

(b) Place each of the letters A, B, C, D and E at its correct position on the probability scale below.



Worksheet 3

1. This table show the entries of some distance running events. Complete the table.

	100m	5000m	Marathon	Total
Male	12			
Female		12	6	
Total	21		21	72

An athlete is chosen at random. What is the probability that the athlete chosen is

	i)	Fema	ale?					-				- 1															
	ii)	Male	mara	athor	n rur	nne	r?									1					1	1		1			
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	iii)	5000	m ruu	nnerî	2																						
	,				•																						٦
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						11																					
2.	The p	robabili	ty sca	ale sł	nows	s th	e pr	ob	ab	iliti	ies	of	the	e ev	'en	ts /	А, В	, C	, D	, ar	nd I	Ξ.					
	\vdash																			Η							
	Α			В				(С					D)					Е							
Which	of the f	five eve	nts:																								
	i)	Has a	n eve	en ch	ance	e of	ha	ppe	eni	ngî	?																
														-											-		
	,																										
	ii)	ls im	possi	ble?																	1		1				7
			_					_																		 _	_
	iii)	ls cer	tain 1	to ha	ppe	n?																					
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				<u> </u>																							
	iv)	ls un	likely	to ha	appe	en?																					
	v)	ls vei	'y like	ely to	hap	per	י?									1					1	1	r —	1		 	

3. There are four red and two yellow marbles in a bag.

One marble is selected at random from the bag. What is the probability the marble selected is:

i) Yellow?

R	ed?)													

4. The probability of a train being late is $\frac{15}{100}$. What is the probability that it will not be late?

- 5. Mary brought a packet of flavoured sweets. There were 24 sweets in the packet: eight apple, five cherry, five orange and the remainder were raspberry.
 - i) How many raspberry sweets were in the packet?

- 6. Mary takes one sweet at random from the packet. What is the probability that the sweet is:
 - ii) Apple flavoured?

iii) Orange or raspberry flavoured?

	-		•	•												
															1 1	1
															1 1	1
																1
																-
															1 1	1
															i	1
															1	

iv) Not cherry flavoured?

v) What two flavours have the same probability of being selected?

- 7. For each of the following events, say whether it is impossible, very unlikely, unlikely, evens, likely, very likely or certain.
 - i) If you buy a lotto ticket you will win the lotto jackpot.

ii) You will get tails when you toss a coin?

iii) You will pick a vowel at random from the letters of the word 'MOUSE'

iv) When a fair die is thrown it will land on a number less than 7.

8. The results from 40 spins of a spinner were recorded as follows.

14	3	2	1	1	2	5	4	4	5	4	3	3	2	1	2	3	3	1
54	3	4	2	1	2	3	5	3	4	3	5	1	4	2	3	4	5	2

i) Complete the following table.

-	-				
Number	1	2	3	4	5
Frequency					

ii) Use the results to estimate the probability of getting a 3 with the next spin.

_												

iii) Is the spinner biased? Give a reason for your answer.

							•								
															1
															1
															1
															1

9. A traffic survey was carried out at a busy intersection for one hour. The results are recorded in the following table.

Direction	Turn Right	Turn Left	Straight Ahead
Number of vehicles	128	108	164

a) How many vehicles were recorded in the survey?

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b) If 5,400 used the intersection during the day, estimate the number of vehicles that turned left.

c) Give one reasons why these figures may not be accurate.

- 10. In a game, two dice are rolled in turn by the players. The table below represents all 36 outcomes. The total on the two dice is recorded.
 - a) Complete the following table, filling in the totals in each case.

			Secon	id dice			
		1	2	3	4	5	6
	1						
First	2		4				
First dice	3						
	4						
	5				9		
	6	7					12

b) State if the following statements are true or false:

Statement	True	False
All totals are equally likely		
A total of 7 is the most likely		
A total of 5 is as likely as a total of 9		
You are not very likely to get a total of 12		
The probability of getting a total of 6 or more is about 50%		
A total of 13 is impossible		
You are more likely to get a total under 7 than over 7		
You get a total of 2 about 3% of the time		

11. A fair coin is flipped twice.

i) Draw a tree diagram to the show all the possible outcomes.

Determine the probability of getting:

ii) Two tails

iii) A tail and a head.

iv) A tail <u>followed</u> by a head

Worksheet 4

1. A biased die is used in a game. The probabilities of getting the six different numbers on the die are shown in the table below.

Number	1	2	3	4	5	6
Probability	0.25	0.25	0.15	0.15	0.1	0.1

Find the expected

value of the random variable X, where X is the number thrown.

2. The table below gives motor insurance information for fully licensed, 17 to 20 year old drivers in Ireland in 2007. All drivers who had their own insurance policy are included.

	Number of drivers	Number of claims	Average cost per claim
Male	9634	977	€6108
Female	6743	581	€6051

a) What is the probability that a randomly selected <u>male</u> driver made a claim during the year? Give your answer correct to three decimal places.

b) What is the probability that a randomly selected **female** driver made a claim during the year?

Give your answer correct to three decimal places.

- 3. The 2006 census shows that the number of females living in Ireland is about the same as the number of males.
 - a) If a person is selected at random, write down the probability that the person is male.

Answer ____

- b) Four people are chosen at random. We are interested in whether they are male or female.
 - i) Complete the sample space below showing the sixteen equally likely outcomes.

MMMM		
MMMF	_	
	-	
	-	
	-	
	-	
	-	

ii) Hence, or otherwise, complete the table of probabilities below.

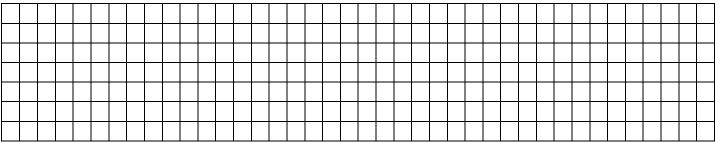
Four males	Three males; One female	Two males; Two females	One male; Three females	Four females
$\frac{1}{16}$				

c) A person states the following: "if you pick four people at random, it's more likely than not that you'll get two males and two females."

Is this statement correct? Justify your answer using the answer(s) to part (b)

Answer_____

Justification:



- 4. On his journey to school Joe's school bus must pass through two sets of traffic lights which are equally likely to be red or green.
 - a) Use a two way table or a tree diagram to show all the possible light combinations for the two sets of lights.

_			, .																	

b) What is the probability that the two lights will be green?

c) When Joe gets to school he likes to spend some time practising free shots in basketball with his friends before class. The probability that he makes a basket is 0.7. What is the probability that in 5 shots he scores his first basket on the fifth shot? Assume all shots are independent events.

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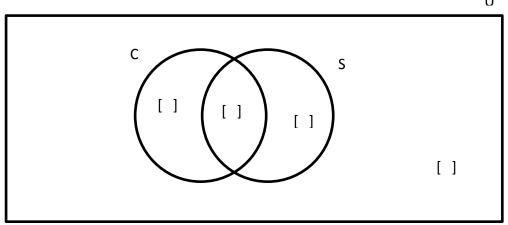
5. A card is selected from an ordinary pack of playing cards. What are the probabilities of it being:

	i)	1	٩b	lacl	k A	ce _			 													
	ii)	1	۹ tv	vo	or a	a th	nre	е														

		iii)	E	Eith	er	an	Ace	e or	a	Kin	g											

There are 13 Clubs in a pack of cards and 4 sevens.

iv) If we call the set of Clubs C and the set of sevens S, complete the Venn diagram below to show the number of elements in each set.



v) If I select a card from an ordinary pack of playing cards, what is the probability that I select:

- i) A club or a seven? _____
- ii) A club and a seven? _____
- iii) A card which is neither a Club nor a seven?

Combinatorics and Probability

Recall Combinatorics are the Fundamenatal Counting Principle (FCP), permutations and combinations.

FCP: Multiply each category of choices by the number of choices.

Permutations: Selecting more than one item without replacement where order is important.

Combinations: Selecting more than one item without replacement where order is not important.

Example 1:

(i) A lottery has 42 numbers from which six are selected. What is the probability of picking all six numbers?

Find n(S). Counting is tedious and impractical, use combinatorics. Selecting more than one item from a group without replacement and order does not matter.

 $\binom{n}{r} = \binom{42}{6} = \frac{42!}{(42-6)!6!} = 5245786 = n(S) \text{ (number of selections)}$

n(picking 6 out of 6) = 1

P(picking 6 out of 6) = 1/5245786

(ii) A lottery has 42 numbers from which six are selected. What is the probability of picking five of the six numbers?

Find n(S). .

$$\binom{n}{r} = \binom{42}{6} = \frac{42!}{(42-6)!6!} = 5245786 = n(S)$$

n(picking 5 out of 6) = $\binom{6}{5}$ $\binom{36}{1}$

P(picking 5 out of 6) =
$$\binom{6}{5} \times \binom{36}{1}$$
 /5245786

Prob:

A lottery has 49 numbers from which six are selected. What is the probability of picking four of the six numbers?

Prob:

A lottery has 42 numbers from which six are selected. What is the probability that two consecutive numbers will be picked.

P (consecutive numbers) = 1 - P(not consecutive)

If none of the six numbers picked are consecutive then the 36 remaining numbers provide 37 slots into which the numbers must slot singly.

! = numbers not picked

 \Box = possible slots for picked numbers

Thus the problem becomes one of selecting 6 slots from a possible 37

The number of ways of doing this is $\binom{37}{6}$

P (no consecutive numbers) =
$$\frac{\binom{37}{6}}{\binom{42}{6}} = \frac{2324784}{5245786} = 0.443$$

P (consecutive numbers)=1- 0.443 = 0.557

Example 2:

Find the probability of being dealt three kings in a five-card hand in a 52-card standard deck. (No jokers) Sample space: how many 5-card hands are there?

Find

$$n(S) = \binom{52}{5} = 2,598,960$$

Find n(3 kings): there are two categories, kings and non-kings.

$$\binom{4}{3} \times \binom{48}{2} = 4 \times 1128 = 4512$$

Notice the sum of the numbers in front of C totals 52 and the number after totals 5. This is one form of bookkeeping and checking your work. n = 52, r = 5

P(3 Kings) =
$$\frac{\binom{4}{3}\binom{48}{2}}{\binom{52}{5}} = \frac{4512}{2,598,960} = 0.001736$$

Example3:

Find the probability of dealing any 3 of a kind.

Since the possibility of 3 kings is .001736, then extend that to any 3 of a kind by multiplying by the number of different possibilities: 13

$$\frac{4512 \times 13}{2,598,960} = .02256$$

Prob:

- 1. Find the probability of being dealt three kings and two aces in a five-card hand in a 52-card standard deck. (No jokers) n(3kings and 2 aces) $\binom{4}{3} \times \binom{4}{2}$
- 2. Find the probability of being dealt a full house in a five-card hand in a 52-card standard deck. (No jokers) $n(full house) = \binom{13}{1} \times \binom{4}{3} \times \binom{12}{1} \times \binom{4}{2} = 3744$
- 3. Find the probability of being dealt two kings and two aces in a five-card hand in a 52-card standard deck. (No jokers) $n(2kings and 2aces) = \binom{4}{2} \times \binom{4}{2} \times \binom{44}{1}$
- 4. Find the probability of being dealt any two pair in a five-card hand in a 52-card standard deck. (No jokers) $n(two pair) = \binom{13}{2} \times \binom{4}{2} \times \binom{44}{1}$
- 5. Find the probability of being dealt five hearts in a five-card hand in a 52-card standard deck. (No jokers) $\begin{pmatrix} 13\\5 \end{pmatrix}$

Find the probability of being dealt five cards of the same suit in a five-card hand in a 52-card standard deck. (No jokers)

LC OL - Exam Questions

Old Course OL Exam Questions

Question 1

A spinner is divided into five equal segments labelled M, A, T, H, S. When tossed, the coin gives either a head or a tail.

The game consists of spinning the arrow, noting the segment that it stops at and then tossing the coin.

For example, one outcome is (M, head).



(a) Complete the sample space below showing the ten possible outcomes of this game.

(ii) F	Answer: Find the p														
	Find the p														
	ind the p	roodonn	ty that s	he aets	a 'hea	d'									
1	•		ty that 5	ne gets	a nea	u.									
	Answer:														
TT1		240													
	inner is s			1				.1 1		() ()	0				
(i) H	How ofter	i would	you exp	ect the	spinne	er to si	op at	the I	etter	•M'	?			7 7	
													_		
				<u> </u>		<u>l</u> [<u> </u>			l		<u> </u>	l
	The actual														

Sarah is on a TV game show called Take the Money and Run.

She has won $\in 10\ 000$ so far. She now has four options:

Option 1: Leave the show with $\in 10,000 - \text{that is}$, Take the Money and Run.

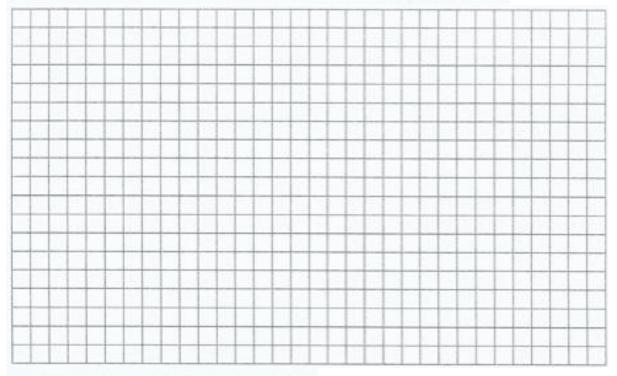
Option 2: Play on and take a 50% chance of winning €50 000

Option 3: Play on and take a 30% chance of winning €75 000

Option 4: Play on and take a 20% chance of winning €100 000

If she plays on and does not win the higher amount, she loses the €10 000.

(a) Calculate the *expected value* of Sarah's winnings for each of the four options.

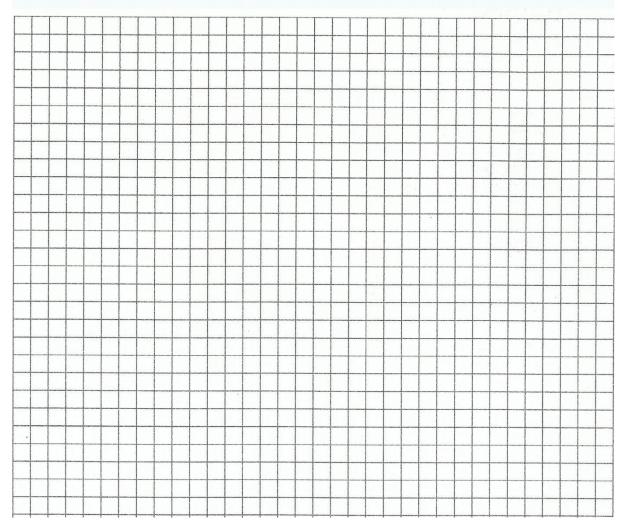


(b) What would you advise Sarah to do, and why?

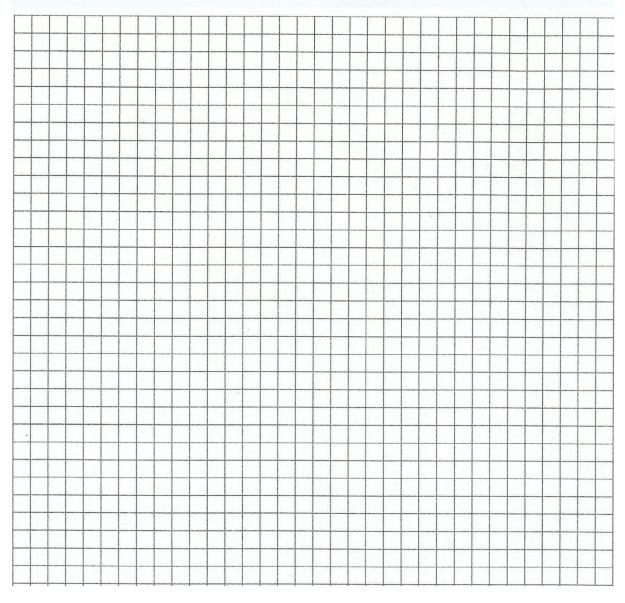


A plastic toy is in the shape of a hemisphere. When it falls on the ground, there are two possible outcomes: it can land with the flat side facing down or with the flat side facing up. Two groups of students are trying to find the probability that it will land with the flat side down.

- (a) Explain why, even though there are two outcomes, the answer is not necessarily equal to $\frac{1}{2}$.
- (b) The students estimate the probability by experiment. Group A drops the toy 100 times. From this, they estimate that it lands flat side down with probability 0.76. Group B drops the toy 500 times. From this, they estimate that it lands flat side down with probability 0.812.
 - (i) Which group's estimate is likely to be better, and why?
 - (ii) How many times did the toy land flat side down for Group B?
 - (iii) Using the data from the two groups, what is the best estimate of the probability that the toy lands flat side down?



- (a) State the *fundamental principle of counting*.
- (b) How many different ways are there to arrange five distinct objects in a row?
- (c) Peter is arranging books on a shelf. He has five novels and three poetry books. He wants to keep the five novels together and the three poetry books together. In how many different ways can he arrange the books?



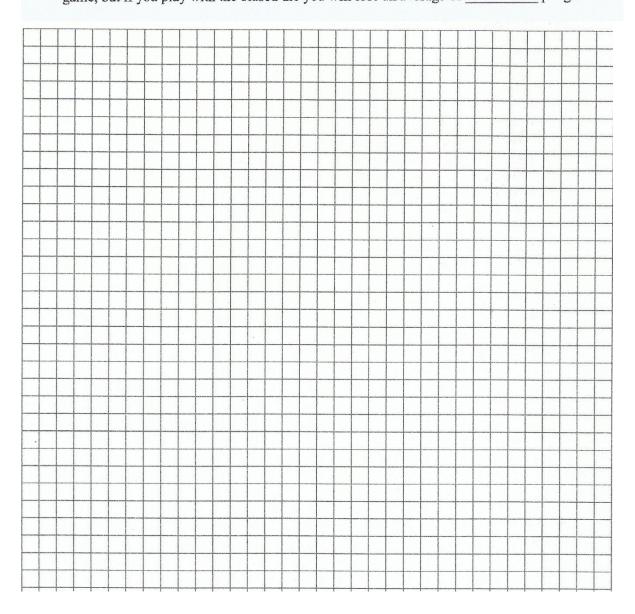
A biased die is used in a game. The probabilities of getting the six different numbers on the die are shown in the table below.

Number	1	2	3	4	5	6
Probability	0.25	0.25	0.15	0.15	0.1	0.1



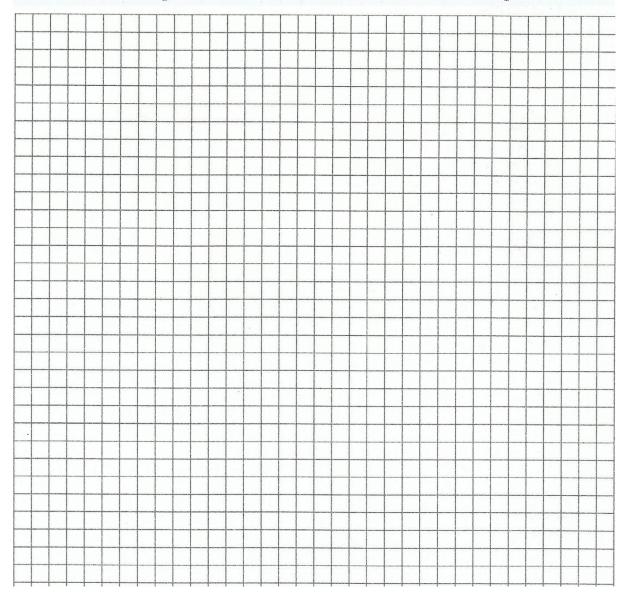
- (a) Find the expected value of the random variable X, where X is the number thrown.
- (b) There is a game at a funfair. It costs €3 to play the game. The player rolls a die once and wins back the number of euro shown on the die. The sentence below describes the difference between using the above biased die and using a fair (unbiased) die when playing this game. By doing the calculations required, complete the sentence.

"If you play the game many times with a fair die, you will win an average of ______ per game, but if you play with the biased die you will lose an average of ______ per game."

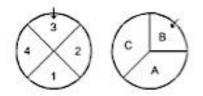


Two spinners, each with four equal segments numbered 1 to 4, are spun.

- (a) Using a list, table, tree diagram, or otherwise, show all the possible outcomes.
- (b) If the spinners are fair, what is the probability of getting two fours?
- (c) Jason thinks that one of the spinners is not fair.Describe an experiment that he could do to find out whether the spinner is fair.

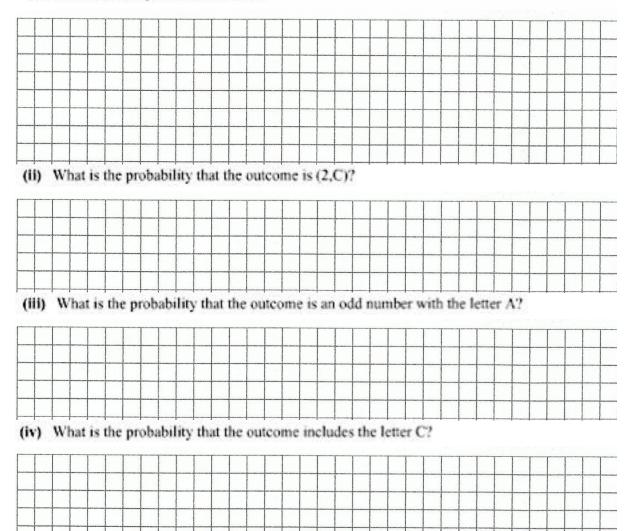


The diagram shows two wheels. The first wheel is divided into 4 equal segments numbered 1, 2, 3, and 4. The second wheel is divided into three equal segments labelled A, B and C. A game consists of spinning the two wheels and noting the segments that stop at the arrows.

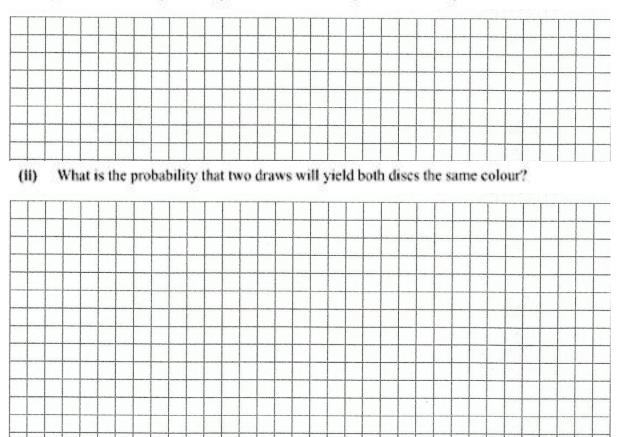


For example, the outcome shown above is (3,B).

(i) Write down all possible outcomes.

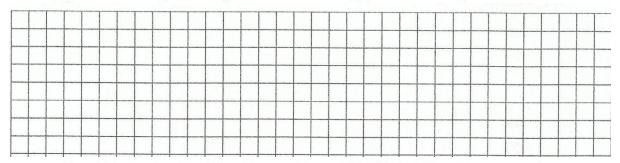


(a) A bag contains 5 red and 3 yellow discs only. When a disc is drawn from the bag, it is returned before the next draw.

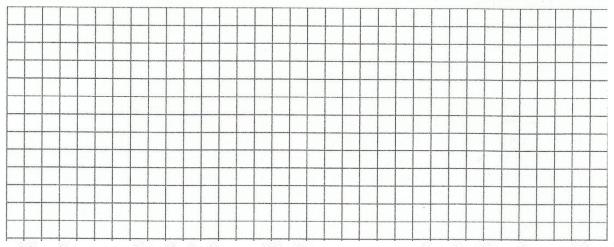


(i) What is the probability that two draws will yield both discs yellow?

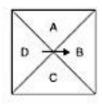
- (a) A bag contains 5 red and 3 yellow discs only. When a disc is drawn from the bag, it is returned before the next draw.
 - (i) What is the probability that two draws will yield both discs yellow?

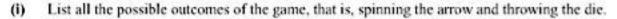


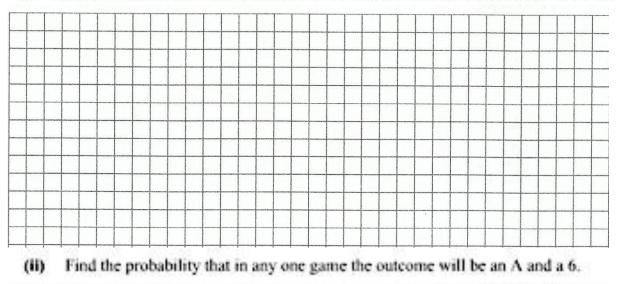
(ii) What is the probability that two draws will yield both discs the same colour?

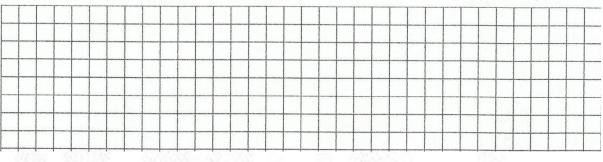


(c) A game consists of spinning an unbiased arrow on a square board and throwing an unbiased die. The board contains the letter A, B, C and D. The board is so designed that when the arrow stops spinning it can only point at one letter and it is equally likely to point at A or B or C or D.









(iii) Find the probability that the outcome is a B and an even number.

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(iv) Find the probability that the outcome is an A or a B and an odd number.

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(v) Find the probability the outcome will be a C or a D and a number ≥ 4 .

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Question 9

Seven people take part in a chess competition. How many games will be played if each person must play each of the others?

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Three cards, numbered 3, 4 and 5 respectively are to be shuffled and placed in a row so that the numbers are visible.

(i) List the six possible outcomes.

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Question 11

(i) How many different arrangements can be made of the letters of the word COMPANY?

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(ii) How many of these begin with the letter C?

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(iii) How many begin with C and end with Y?

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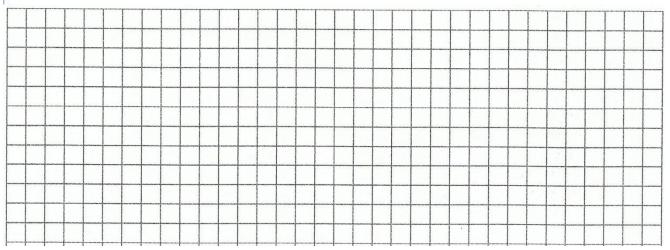
Question 12

A bag contains 10 green marbles and three red ones. Two marbles are drawn at random but not replaced.

(i) Construct a tree diagram to show the outcomes.

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(ii) Find the probability that they will have a different colour.



A bag contains 10 green marbles and three red ones. Two marbles are drawn at random but not replaced.

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(i) Construct a tree diagram to show the outcomes.

#### (ii) Find the probability that they will have a different colour.

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#### **Question 14**

A code consists of a four-digit number which is formed from the digits 3 to 9 inclusive. No digit can occur more than once in the code.

(i) How many different codes are possible?

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#### (ii) How many of the four-digit codes are greater than 6,000?

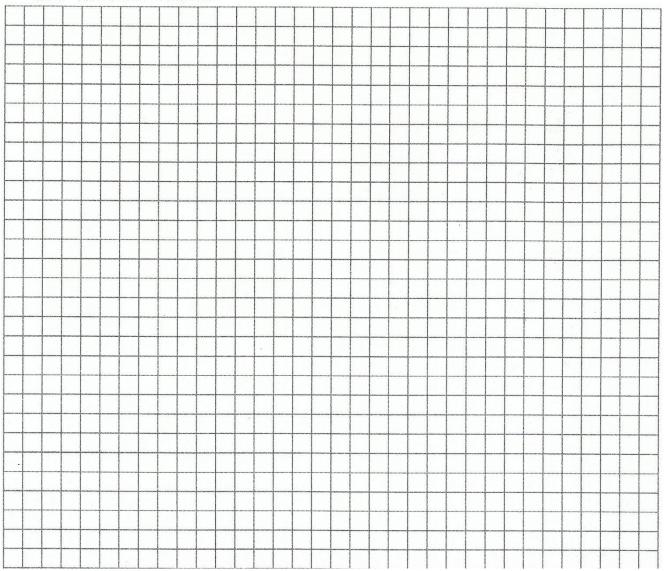
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(iii) How many of the four-digit codes are divisible by 2?

#### **Question 15**

(b) Three cards are chosen at random from a standard pack of playing cards. Find the probability that:

- (i) All three cards are Kings.
- (ii) None of the three cards is a King.
- (iii) At least one is a King.
- (iv) Exactly two are Clubs.



The letters in the word FERMAT are arranged taking all the letters each time. How many different arrangements are possible if:

#### (i) There are no restrictions.

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### (ii) The arrangements begin with the letter F and end with a vowel.

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#### (iii) The two vowels are together.

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#### **Question 17**

There are eight questions on an examination paper.

#### (i) In how many different ways can a candidate select six questions?

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#### (ii) In how many different ways can a candidate select six questions if one particular question must always be selected?

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A letter is selected at random from the letters of the word COMPANION.

(i) Find the probability that the letter is O.

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#### (ii) Find the probability that the letter is M or N.

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(iii) Find the probability that the letter is a vowel.

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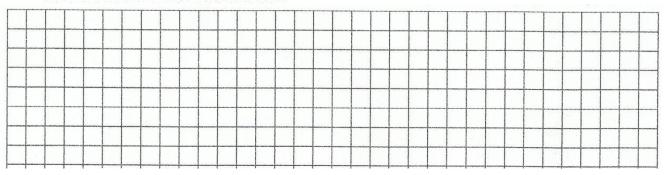
## **Question 19**

A team of 5 players is to be chosen from 8 players.

(i) In how many ways can the 5 players be chosen if all are eligible for selection?

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(ii) If the 8 players include a goalkeeper, in how many ways may a team of 5 be selected if each team must include the goalkeeper?



John takes three coins from his pocket and flips the three coins together. He repeats the experiment 25 times and records his results as follows.

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тнн	TTT	HTT	THT	ннн

(i) Copy and complete the frequency table.

Result	3 Heads	2 Heads	1 Head	0 Heads
Frequency				S

(ii) What percentage of the throws revealed one head only?

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(iii) Use an appropriate graph to display the data.

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There are 40 people in a club, 24 male and 16 female. Four of the males and two of the females wear glasses.

(i) When a club member is selected at random, what is the probability that the person is a male?

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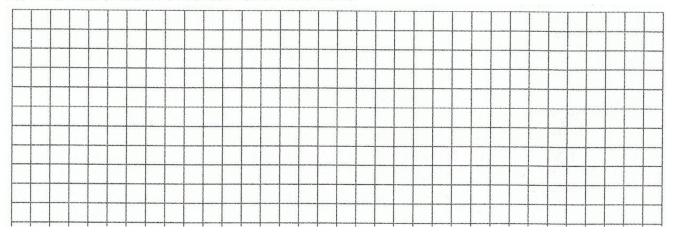
#### (ii) What is the probability that the person is a male not wearing glasses?

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# (i) How many different four-digit numbers can be formed from the digits 2, 4, 6 and 8, if no digit can be used more than once in a number?

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(ii) How many of these numbers are greater than 6,000?



(iii) How many of these numbers end with 8?

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