Proof by Contradiction

A proof by contradiction is a proof in which one assumption is made. Then, by using valid arguments, a statement is arrived at which is clearly false, so the original assumption must have been false.

Possible Answers:

Explanation:

To show a proposition is true, assume its opposite is true
and then show this leads to a conclusion that is false.
Thus, the original proposition is true because its opposite is false.

[It is assumed that a proposition is either true or false.]

Example:

To prove that $\sqrt{2}$ is irrational (cannot be written as a fraction).

Suppose that $\sqrt{2} = \frac{m}{n}$ for integers m and n, $n \neq 0$, and that the fraction $\frac{m}{n}$ is such that m and n have a highest common factor of 1.

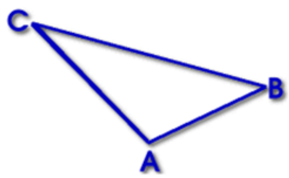
We can then show that both m and n must be even. That is, both m and n are divisible by 2.

But this is false, as m and n have a highest common factor of 1.

So $\sqrt{2}$ cannot be written as fraction, and the proof is complete.

Geometry : Proof by Contradiction

Triangle ABC has no more than one right angle. Can you complete a proof by contradiction for this statement?



Assume ∠A and ∠B are right angles

We know ∠A + ∠B + ∠C = 180^0

By substitution $90^0 + 90^0 + ∠C = 180^0$

∴ ∠C = 0^0 which is a contradiction

∴ ∠A and ∠B cannot both be right angles

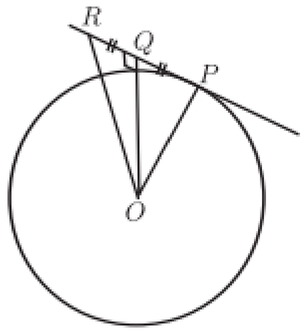
⇒ A triangle can have at most one right angle

‘Tangent’ Method

Theorem 20: Each tangent is perpendicular to the radius that goes to the point of contact.

Suppose the point of contact is P and the tangent l is not perpendicular to OP.

- Let the perpendicular to the tangent from the centre O meet it at Q.
- Pick R on P Q, on the other side of Q from P, with |QR| = |P Q| (as below).



Then ΔOQR is congruent to ΔOQP. [SAS]

∴ |OR| = |OP|, so R is a second point where l meets the circle. This contradicts the given fact that l is a tangent.

Thus l must be perpendicular to OP, as required.