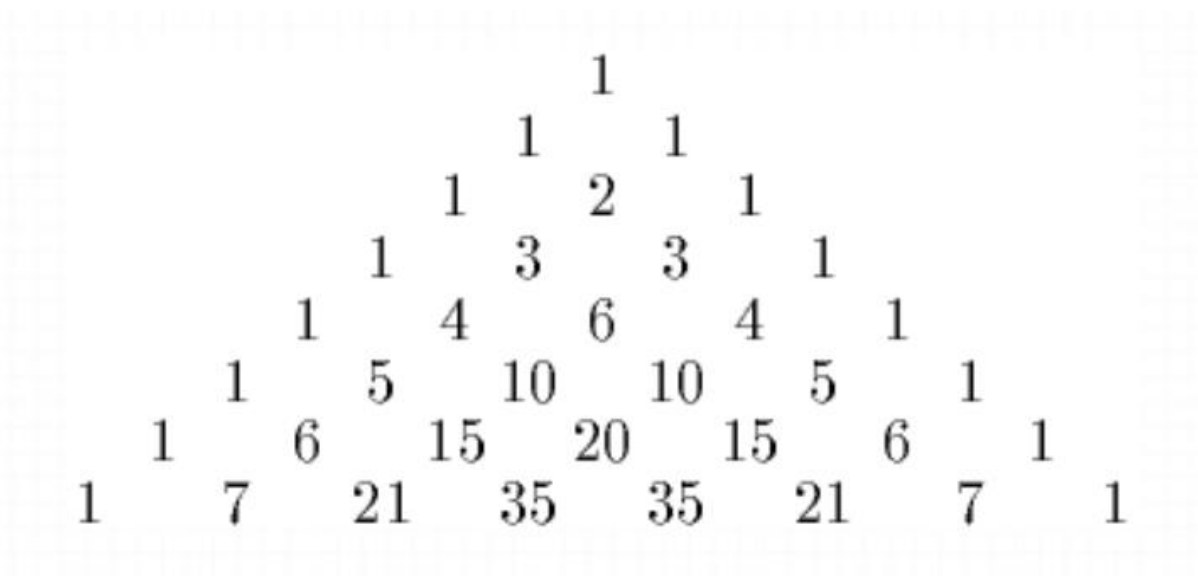# Binomial Theorem

You need to be able:

- Expand expressions $(p + q)^n$ using the binomial theorem
- To see a pattern in the expansion
- Write down the middle term, a particular term or the general term
- Use the binomial theorem as part of other algebraic questions

Pascal's Triangle

- In previous algebra questions you would have simply multiplied out various questions that were given in the form of $(p + q)^n$
- The coefficient of each term can be found using the below triangle

$$
\begin{array}{ccccccccccccc}
 & & & & & & 1 & & & & & & \\
 & & & & & 1 & & 1 & & & & & \\
 & & & & 1 & & 2 & & 1 & & & & \\
 & & & 1 & & 3 & & 3 & & 1 & & & \\
 & & 1 & & 4 & & 6 & & 4 & & 1 & & \\
 & 1 & & 5 & & 10 & & 10 & & 5 & & 1 & \\
1 & & 6 & & 15 & & 20 & & 15 & & 6 & & 1 \\
\end{array}
$$

$$1 \quad 7 \quad 21 \quad 35 \quad 35 \quad 21 \quad 7 \quad 1$$

Eg. 1 & 2

$$(x + y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$$

$$(a + 2)^3 = a^3 + 3a^2(2) + 3a(2)^2 + (2)^3$$
$$= a^3 + 6a^2 + 12a + 8$$

Eg 3

$$(t - 2)^5$$

$$= t^5 + 5t^4(-2) + 10t^3(-2)^2 + 10t^2(-2)^3 + 5t(-2)^4 + (-2)^5$$

$$= t^5 - 10t^4 + 40t^3 - 80t^2 + 80t - 32$$

Eg 4

$$(2x + 3)^3 = (2x)^3 + 3(2x)^2(3) + 3(2x)(3)^2 + (3)^3$$

$$= 8x^3 + 36x^2 + 54x + 27$$

General Formula for Expansion (Binomial Theorem)

- A faster system can be used to expand equations in the form of $(p + q)^n$
- This formula can be found on Pg. 20 of the Formulae and Tables Booklet

$$(p + q)^n = \binom{n}{0} p^n q^0 + \binom{n}{1} p^{n-1} q^1 + \binom{n}{2} p^{n-2} q^2 + \cdots + \binom{n}{n-1} p^1 q^{n-1} \binom{n}{n} p^0 q^n$$

- $\binom{n}{0}$ or $\binom{n}{r}$ means $.^n C_r$, n choose r, which is a function in your calculator.

Eg. 5

$$\left(x + y\right)^7$$

$$= \binom{7}{0} x^7 y^0 + \binom{7}{1} x^6 y^1 + \binom{7}{2} x^5 y^2 + \binom{7}{3} x^4 y^3 + \binom{7}{4} x^3 y^4$$

$$+ \binom{7}{5} x^2 y^5 + \binom{7}{6} x^1 y^6 + \binom{7}{7} x^0 y^7$$

$$= x^7 + 7x^6 y + 21x^5 y^2 + 21x^5 y^2 + 35x^3 y^4 + 21x^2 y^5 + 7x^1 y^6 + y^7$$

In Class Examples

Eg. 6
Find the x^3

$$(1 + x)^{10}$$

Eg. 7
Expand fully

$$(1 - 3x)^4$$

Eg. 8
Expand fully

$$(2 - 5x)^5$$

Eg. 9
Find the coefficient of a^4

$$(3 - 2a)^{10}$$

Finding particular terms in an expansion

- To find the coefficient of a particular term you can speed up the process by:

$$(p + q)^n$$

Find: q^r

$$\binom{n}{r} p^{n-r} q^r$$

Eg. 9

Find the coefficient of a^4

$$(3 - 2a)^{10}$$

Eg. 10

Find the middle term in expansion of

$$\left(\frac{2}{x} - x^2\right)^6$$

The power of 6 would mean that there will be 7 TERMS so the middle term should be the fourth term…. r=3

Eg. 11

The coefficient of x^4 is 240 in the binomial expansion of $(1 + kx)^6$ find the two values of k

Homework Questions

1) Coefficient of n^2 in expansion of $(n + 2)^5$

2) Coefficient of x^3 in expansion of $(3 + x)^4$

3) Coefficient of b^2 in expansion of $(b + 3)^4$

4) Coefficient of y^2 in expansion of $(2 + y)^4$

5) Coefficient of u^2 in expansion of $(u + 4)^4$

6) Coefficient of y^3 in expansion of $(4 + y)^4$

7) 5th term in expansion of $(x + 3)^5$

8) 2nd term in expansion of $(x + 4)^4$

9) 3rd term in expansion of $(v + 2)^4$

10) 3rd term in expansion of $(4 + b)^4$

11) 3rd term in expansion of $(2 + n)^5$

12) 2nd term in expansion of $(3 + u)^4$

13) $(y + 2)^4$

14) $(x + 4)^4$

15) $(b + 3)^5$

16) $(2 + a)^4$

17) $(3 + x)^5$

18) $(4 + a)^4$

19) Coefficient of x^2y in expansion of $(x - y)^3$

20) Coefficient of u in expansion of $(u - 3)^4$

21) Coefficient of y^2 in expansion of $(5 + y)^3$

22) Coefficient of x^2y^2 in expansion of $(x + y)^4$

23) Coefficient of u^2 in expansion of $(u - 2)^3$

24) Coefficient of y in expansion of $(y - 5)^3$

25) Coefficient of y^3x in expansion of $(y - x)^4$

26) Coefficient of a^2 in expansion of $(a - 3)^3$

27) Coefficient of n^2 in expansion of $(3n - 1)^4$

28) Coefficient of xy^2 in expansion of $(2x + 5y)^3$

29) Coefficient of m^2n^2 in expansion of $(3m + n)^4$

30) Coefficient of y^2 in expansion of $(2y + 1)^3$

31) Coefficient of m^2n in expansion of $(m + 2n)^3$

32) Coefficient of nm^2 in expansion of $(n - 3m)^3$

33) Coefficient of y^2x^2 in expansion of $(y + 4x)^4$

34) Coefficient of m^2 in expansion of $(3m + 1)^4$

35) Coefficient of x^3 in expansion of $(2x - 1)^4$

36) Coefficient of yx^2 in expansion of $(4y + 2x)^3$

37) Coefficient of ab^3 in expansion of $(a - 3b)^4$

38) Coefficient of m^2n in expansion of $(m + 5n)^3$

39) Coefficient of n^2m^2 in expansion of $(3n - 4m)^4$

40) Coefficient of nm^2 in expansion of $(2n + 3m)^3$

41) Coefficient of nm^2 in expansion of $(n - 5m)^3$

42) Coefficient of xy^3 in expansion of $(2x + y)^4$

43) 1st term in expansion of $(x - 5)^3$

44) 2nd term in expansion of $(4 - x)^4$

45) 2nd term in expansion of $(y + 2)^4$

46) 3rd term in expansion of $(y + 3)^4$

47) 2nd term in expansion of $(2 - y)^3$

48) 1st term in expansion of $(x + 4)^3$

49) 1st term in expansion of $(a - 4)^4$

50) 1st term in expansion of $(4y + 1)^3$

51) 4th term in expansion of $(3 + b)^3$

52) 1st term in expansion of $(3x + 1)^4$

53) 1st term in expansion of $(b - 5)^3$

54) 2nd term in expansion of $(2y - 1)^4$

55) 2nd term in expansion of $(m - 4)^4$

56) 3rd term in expansion of $(y + x)^3$

57) 2nd term in expansion of $(2y + 1)^4$

58) 3rd term in expansion of $(5y + 3x)^3$

59) 2nd term in expansion of $(4x - 1)^3$

60) 2nd term in expansion of $(v - 3u)^4$

61) 4th term in expansion of $(x + 3y)^4$

62) 3rd term in expansion of $(u + v)^3$

63) 5th term in expansion of $(u - v)^4$

64) 4th term in expansion of $(3y + x)^3$

65) 1st term in expansion of $(2x - 4y)^3$

66) 4th term in expansion of $(x - 4y)^3$

67) $(u - v)^3$

68) $(u + v)^4$

69) $(m + 2n)^4$

70) $(y + x)^4$

71) $(2 + b^2)^4$

72) $(4a^3 + 1)^4$

73) $(x^3 + y)^7$

74) $(4x + 1)^2$

75) $(y + 4)^2$

76) $(m + n)^5$

77) $(5y^4 - x)^3$

78) $(2x^4 - 1)^6$

79) $(2y + 1)^6$

80) $(x^4 - y^3)^3$

81) $(x + 2)^7$

82) $(x - y)^4$

83) $(2u^4 + 1)^4$

84) $(x + 2y)^7$

85) $(y - 3x^2)^2$

86) $(3u - 1)^5$

87) $(x - 9y)^2$

88) $(1 - 2b^4)^5$

89) $(2y - x^4)^6$

90) $(5u^2 - 1)^3$

91) $(3v^4 + 1)^3$

92) $(y^4 + 2x)^7$

93) $(1 + 4x^4)^4$

94) $(2y^2 + x)^7$

95) $(6y^4 - 1)^2$

96) $(3v^4 - 1)^4$

97) $(10y^3 + 1)^2$

98) $(2y^3 + x)^5$

99) $(y^3 - 2x^4)^5$

100) $(5m^3 - 1)^3$

Answers

1) 80

2) 12

3) 54

4) 24

5) 96

6) 16

7) $405x$

8) $16x^3$

9) $24v^2$

10) $96b^2$

11) $80n^2$

12) $108u$

13) $y^4 + 8y^3 + 24y^2 + 32y + 16$

14) $x^4 + 16x^3 + 96x^2 + 256x + 256$

15) $b^5 + 15b^4 + 90b^3 + 270b^2 + 405b + 243$

16) $16 + 32a + 24a^2 + 8a^3 + a^4$

17) $243 + 405x + 270x^2 + 90x^3 + 15x^4 + x^5$

18) $256 + 256a + 96a^2 + 16a^3 + a^4$

19) -3

20) -108

21) 15

22) 6

23) -6

24) 75

25) -4

26) -9

27) 54

28) 150

29) 54

30) 12

31) 6

32) 27

33) 96

34) 54

35) -32

36) 48

37) -108

38) 15

39) 864

40) 54

41) 75

42) 8

43) x^3

44) $-256x$

45) $8y^3$

46) $54y^2$

47) $-12y$

48) x^3

49) a^4

50) $64y^3$

51) b^3

52) $81x^4$

53) b^3

54) $-32y^3$

55) $-16m^3$

56) $3yx^2$

57) $32y^3$

58) $135yx^2$

59) $-48x^2$

60) $-12v^3u$

61) $108xy^3$

62) $3uv^2$

63) v^4

64) x^3

65) $8x^3$

66) $-64y^3$

67) $u^3 - 3u^2v + 3uv^2 - v^3$

68) $u^4 + 4u^3v + 6u^2v^2 + 4uv^3 + v^4$

69) $m^4 + 8m^3n + 24m^2n^2 + 32mn^3 + 16n^4$

70) $y^4 + 4y^3x + 6y^2x^2 + 4yx^3 + x^4$

71) $16 + 32b^2 + 24b^4 + 8b^6 + b^8$

72) $256a^{12} + 256a^9 + 96a^6 + 16a^3 + 1$

73) $x^{21} + 7x^{18}y + 21x^{15}y^2 + 35x^{12}y^3 + 35x^9y^4 + 21x^6y^5 + 7x^3y^6 + y^7$

74) $16x^2 + 8x + 1$

75) $y^2 + 8y + 16$

76) $m^5 + 5m^4n + 10m^3n^2 + 10m^2n^3 + 5mn^4 + n^5$

77) $125y^{12} - 75y^8x + 15y^4x^2 - x^3$

78) $64x^{24} - 192x^{20} + 240x^{16} - 160x^{12} + 60x^8 - 12x^4 + 1$

79) $64y^6 + 192y^5 + 240y^4 + 160y^3 + 60y^2 + 12y + 1$

80) $x^{12} - 3x^8y^3 + 3x^4y^6 - y^9$

81) $x^7 + 14x^6 + 84x^5 + 280x^4 + 560x^3 + 672x^2 + 448x + 128$

82) $x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4$

83) $16u^{16} + 32u^{12} + 24u^8 + 8u^4 + 1$

84) $x^7 + 14x^6y + 84x^5y^2 + 280x^4y^3 + 560x^3y^4 + 672x^2y^5 + 448xy^6 + 128y^7$

85) $y^2 - 6yx^2 + 9x^4$

86) $243u^5 - 405u^4 + 270u^3 - 90u^2 + 15u - 1$

87) $x^2 - 18xy + 81y^2$

88) $1 - 10b^4 + 40b^8 - 80b^{12} + 80b^{16} - 32b^{20}$

89) $64y^6 - 192y^5x^4 + 240y^4x^8 - 160y^3x^{12} + 60y^2x^{16} - 12yx^{20} + x^{24}$

90) $125u^6 - 75u^4 + 15u^2 - 1$

91) $27v^{12} + 27v^8 + 9v^4 + 1$

92) $y^{28} + 14y^{24}x + 84y^{20}x^2 + 280y^{16}x^3 + 560y^{12}x^4 + 672y^8x^5 + 448y^4x^6 + 128x^7$

93) $1 + 16x^4 + 96x^8 + 256x^{12} + 256x^{16}$

94) $128y^{14} + 448y^{12}x + 672y^{10}x^2 + 560y^8x^3 + 280y^6x^4 + 84y^4x^5 + 14y^2x^6 + x^7$

95) $36y^8 - 12y^4 + 1$

96) $81v^{16} - 108v^{12} + 54v^8 - 12v^4 + 1$

97) $100y^6 + 20y^3 + 1$

98) $32y^{15} + 80y^{12}x + 80y^9x^2 + 40y^6x^3 + 10y^3x^4 + x^5$

99) $y^{15} - 10y^{12}x^4 + 40y^9x^8 - 80y^6x^{12} + 80y^3x^{16} - 32x^{20}$

100) $125m^9 - 75m^6 + 15m^3 - 1$