Algebra 3

Inequalities

- Solve linear and quadratic inequalities
- Solve rational and modulus inequalities
- Graph a solution set on a number line
- Solve abstract inequalities

Some Rules

1. *Multiplying or dividing an inequality by a negative number reverses the in the direction of the inequality symbol*
2. $x \in N$ or $x \in Z$ *use dots when writing on a number line*
   $x \in R$ *use a heavy line when plotting on a number line*

Linear Inequalities

Eg 1

Find the solution set of

$$11 - 3x \geq 2$$

and graph on a number line $x \in N$.

Eg 2

i. Find the solution set A of $2x + 7 \leq 11, x \in R$

ii. Find the solution set B of $4 - 2x < 10, x \in R$

iii. Find $A \cap B$ and graph on a number line

Rational Equalities

- Just multiply both sides of the equation by the square of the denominator

Eg. 1

Solve the inequality

$$\frac{3x + 1}{x - 1} \geq 2$$

Modulus Inequalities

- Remember that the modulus of a number is the positive value of that number

$$|x| \leq a \; then - a \leq x \leq a$$
$$|x| \geq a, then\, x \leq -a \; or \; x \geq a$$

Eg. 1

Solve the following inequality

$$|5 - 2x| \leq 3$$

Eg. 2

Solve the following inequality

$$|4x + 7| \geq 1$$

Abstract Inequalities

1. Write down the inequality to be proved
2. Using reversible steps for inequalities make a true algebraic inequality
3. If the above is true then the inequality must be true

Remember

$$(real\ number)^2 \geq 0, and - (real\ number)^2 \leq 0$$

Eg. 1

Prove that the following is true

$$a^2 + 4b^2 \geq 4ab$$

Eg. 2

Prove the following

$$\frac{a}{b^2} + \frac{b}{a^2} \geq \frac{1}{a} + \frac{1}{b}$$

Eg. 3

Prove the following has real roots

$$x^2 - 4px - x + 2p = 0$$

Indices

- To know and apply rules of indices
- Solve problems involving indices

Basic Indices

The first rule: $a^n \times a^m = a^{m+n}$

The second rule: $(a^n)^m = a^{mn}$

The third rule: $a^m \div a^n = a^{m-n}$

The fourth rule: $a^0 = 1$

The fifth rule: $a^{-1} = \frac{1}{a}$ $\quad a^{-m} = \frac{1}{a^m}$

The sixth rule: $a^{½} = \sqrt{a}$ $\quad a^{\frac{1}{m}} = \sqrt[m]{a}$

$$a^{\frac{n}{m}} = (a^{\frac{1}{m}})^n = (\sqrt[m]{a})^n$$

Eg. 1

Solve the following

$125^{\frac{2}{3}}$	$32^{\frac{2}{5}}$	$\frac{4^{-\frac{1}{2}}}{64^{\frac{2}{3}}}$

Exponential Equations

1. Rewrite given equation to the same power
2. Bring everything to the same base
3. Equate the powers to be equal, solve for x

Eg.1

Solve for x in the following

$$27^{4+3x} = 243^{1+2x}$$

Eg. 2

Solve for x;

$$2^{x^2} = 8^{2x+9}$$

Eg. 3

Solve for x and y in the following

$$2^{x} = 8^{y+1}$$

$$3^{x-9} = 9^{y}$$

Solving by Substitution

Eg. 1

Solve the equation

$$2^{2x+1} - 5(2^{x}) + 2 = 0$$

Eg.2

Solve the equation

$$3^{x+2} - 82 + 3^{2-x} = 0$$

Logarithms

- To know and apply the rules of logs
- Solve problems involving logs

General Rule of Logs

$$a = b^c \equiv \log_b a = c$$

Eg.1

Evaluate the following

$$\log_7 343$$

Eg. 2

Evaluate the following

$$\log_{27} \frac{1}{3}$$

Natural Logs

$\log_b(x \cdot y) = \log_b(x) + \log_b(y)$	
$\log_b(x / y) = \log_b(x) - \log_b(y)$	
$\log_b(x^y) = y \cdot \log_b(x)$	
$\log_b(c) = 1 / \log_c(b)$	$\log_b(1) = 0$
$\log_b(x) = \log_c(x) / \log_c(b)$	$\log_b(b) = 1$

Log Equations

1. Equate to a single log on both sides, equate LHS=RHS

Or

2. Get a single log and change to index form

NOTE: Ensure all logs are to the same base

Eg. 1

Solve

$$\log_2(x + 6) - \log_2(x + 2) = 1$$

Eg. 2

$$\log_e(x+1) + \log_e(x-1) = \log_e 3$$

Eg. 3

$$\log_2 x - \log_2(x-1) = 4\log_4 2$$