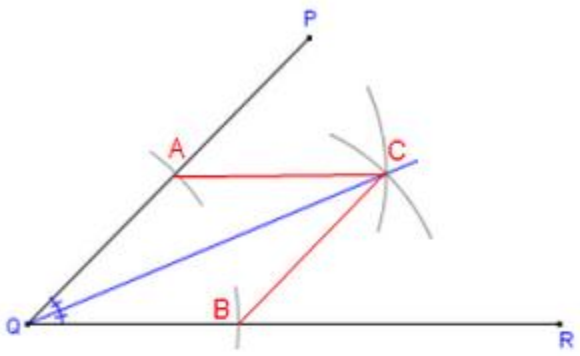
The Prescribed Constructions

No	Construction		Done
1.	Bisector of a given angle, using only compass and straight edge.	HL	
2.	Perpendicular bisector of a segment, using only compass and straight edge.	HL	
3.	Line perpendicular to a given line l, passing through a given point not on l.	HL	
4.	Line perpendicular to a given line l, passing through a given point on l.	HL	
5.	Line parallel to given line, through given point.	HL	
6.	Division of a segment into 2, 3 equal segments, without measuring it.	HL	
7.	Division of a segment into any number of equal segments, without measuring it.	HL	
8.	Line segment of given length on a given ray.	HL	
9.	Angle of given number of degrees with a given ray as one arm.	HL	
10.	Triangle, given lengths of three sides.	HL	
11.	Triangle, given SAS data.	HL	
12.	Triangle, given ASA data.	HL	
13.	Right-angled triangle, given the length of the hypotenuse and one other side.	HL	
14.	Right-angled triangle, given one side and one of the acute angles (several cases).	HL	
15.	Rectangle, given side lengths.	HL	
16.	Circumcentre and circumcircle of a given triangle, using only straightedge and compass.	OL	
17.	Incentre and incircle of a given triangle, using only straight-edge and compass.	OL	
18.	Angle of 60°, without using a protractor or set square.	FL	
19.	Tangent to a given circle at a given point on it.	FL	
20.	Parallelogram, given the length of the sides and the measure of the angles.	FL	
21.	Centroid of a triangle.	OL	
22.	Orthocentre of a triangle.	HL	

Bisector of a given angle, using only compass and straight edge.

This construction works by effectively building two congruent triangles. The image below is the final drawing above with the red lines added and points A,B,C labelled.



Start with angle PQR that we will bisect.

1. Place the compasses' point on the angle's vertex Q.

2. Adjust the compasses to a medium wide setting. The exact width is not important.

3. Without changing the compasses' width, draw an arc across each leg of the angle.

4. The compasses' width can be changed here if desired. Recommended: leave it the same.

5. Place the compasses on the point where one arc crosses a leg and draw an arc in the interior of the angle.

7. Using a straightedge or ruler, draw a line from the vertex to the point where the arcs cross

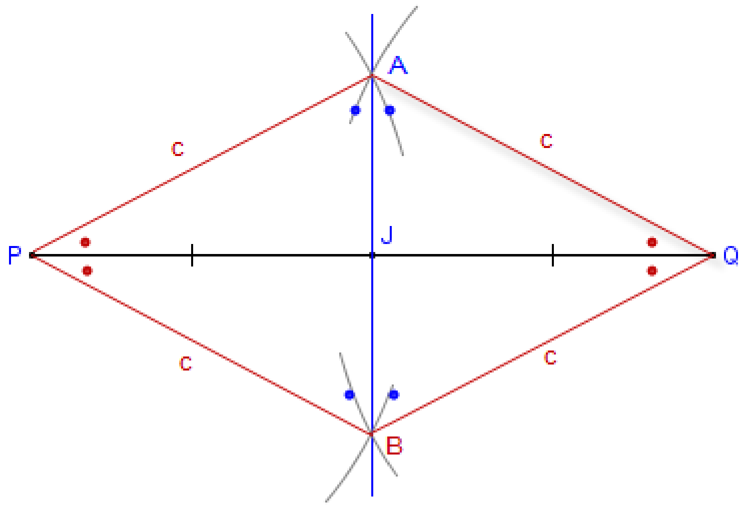
Done. This is the bisector of the angle ∠PQR.

	Argument	Reason
1	QA is congruent to QB	They were both drawn with the same compass width
2	AC is congruent to BC	They were both drawn with the same compass width
3	∆QAC and ∆QBC are congruent	Three sides congruent (sss). QC is common to both.
4	Angles AQC, BQC arecongruent	CPCTC. Corresponding parts of congruent triangles are congruent
5	The line QC bisects the angle PQR	Angles AQC, BQC are adjacent and congruent

Perpendicular bisector of a segment, using only compass and straight edge.

This construction works by effectively building congruent triangles that result in right angles being formed at the midpoint of the line segment. The proof is surprisingly long for such a simple construction.

The image below is the final drawing above with the red lines and dots added to some angles.



Start with a line segment PQ.   Geometry construction with compass and straightedge or ruler or ruler

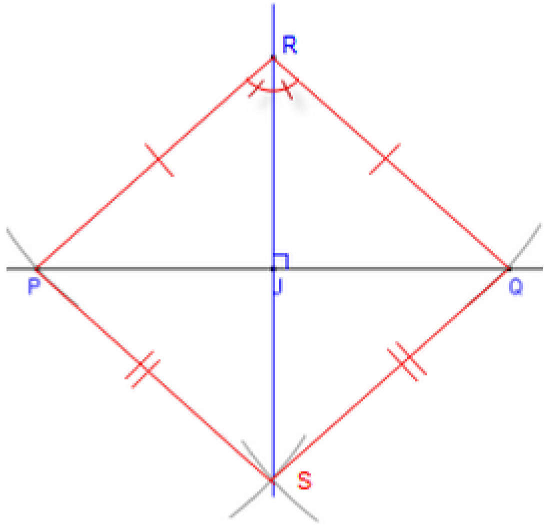
1. Place the compasses on one end of the line segment
2. Set the compasses' width to a approximately two thirds the line length. The actual width does not matter.
3. Without changing the compasses' width, draw an arc above and below the line.
4. Again without changing the compasses' width, place the compasses' point on the other end of the line. Draw an arc above and below the line so that the arcs cross the first two.
5. Using a straightedge, draw a line between the points where the arcs intersect.

Done. This line is perpendicular to the first line and bisects it (cuts it at the exact midpoint of the line).

	Argument	Reason
1	Line segments AP, AQ, PB, QB are all congruent	The four distances were all drawn with the same compass width c.
Next we prove that the top and bottom triangles are isosceles and congruent		
2	Triangles ΔAPQ and ΔBPQ are isosceles	Two sides are congruent (length c)
3	Angles AQJ, APJ are congruent	Base angles of isosceles triangles are congruent
4	Triangles ΔAPQ and ΔBPQ are congruent	Three sides congruent (sss). PQ is common to both.
5	Angles APJ, BPJ, AQJ, BQJ are congruent. (The four angles at P and Q with red dots)	CPCTC. Corresponding parts of congruent triangles are congruent
Then we prove that the left and right triangles are isosceles and congruent		
6	ΔAPB and ΔAQB are isosceles	Two sides are congruent (length c)
7	Angles QAJ, QBJ are congruent.	Base angles of isosceles triangles are congruent
8	Triangles ΔAPB and ΔAQB are congruent	Three sides congruent (sss). AB is common to both.

	Argument	Reason
9	Angles PAJ, PBJ, QAJ, QBJ are congruent. (The four angles at A and B with blue dots)	CPCTC. Corresponding parts of congruent triangles are congruent
	Then we prove that the four small triangles are congruent and finish the proof	
10	Triangles ΔAPJ, ΔBPJ, ΔAQJ, ΔBQJ arecongruent	Two angles and included side (ASA)
11	The four angles at J - AJP, AJQ, BJP, BJQ arecongruent	CPCTC. Corresponding parts of congruent triangles are congruent
12	Each of the four angles at J are 90°. Therefore AB is perpendicular to PQ	They are equal in measure and add to 360°
13	Line segments PJ and QJ are congruent. Therefore AB bisects PQ.	From (8), CPCTC. Corresponding parts of congruent triangles are congruent

Line perpendicular to a given line l, passing through a given point not on l.



Start with a line and point R which is not on that line. Geometry construction with compass and straightedge or ruler or ruler or ruler

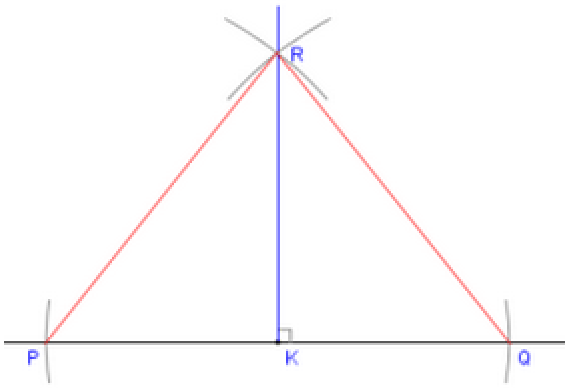
1. Place the compasses on the given external point R.
2. Set the compasses' width to a approximately 50% more than the distance to the line. The exact width does not matter.
3. Draw an arc across the line on each side of R, making sure not to adjust the compasses' width in between. Label these points P and Q
4. From each point P, Q, draw an arc below the line so that the arcs cross.
5. Place a straightedge between R and the point where the arcs intersect. Draw the perpendicular line from R to the line, or beyond if you wish.

Done. This line is perpendicular to the first line and passes through the point R. It also bisects the segment PQ (divides it into two equal parts)

	Argument	Reason
1	Segment RP is congruent to RQ	They were both drawn with the same compass width
2	Segment SQ is congruent SP	They were both drawn with the same compass width
3	Triangle RQS is congruent to triangle RPS	Three sides congruent (sss), RS is common to both.
4	Angle JRQ is congruent to JRP	CPCTC. Corresponding parts of congruent triangles are congruent.
5	Triangle RJQ is congruent to triangle RJP	Two sides and included angle congruent (SAS), RJ is common to both.
6	Angle RJP and RJQ are congruent	CPCTC. Corresponding parts of congruent triangles are congruent.
7	Angl RJP and RJQ are 90°	They are congruent and supplementary (add to 180°).

Line perpendicular to a given line l, passing through a given point on l.

This construction works by effectively building two congruent triangles. The image below is the final drawing above with the red lines added.



Start with a line and point K on that line. Geometry construction with compass and straightedge or ruler or ruler

1. Set the compasses' width to a medium setting. The actual width does not matter.
2. Without changing the compasses' width, mark a short arc on the line at each side of the point K, forming the points P,Q. These two points are thus the same distance from K.
3. Increase the compasses to almost double the width (again the exact setting is not important).
4. From P, mark off a short arc above K
5. Without changing the compasses' width repeat from the point Q so that the the two arcs cross each other, creating the point R
6. Using the straight edge, draw a line from K to where the arcs cross.

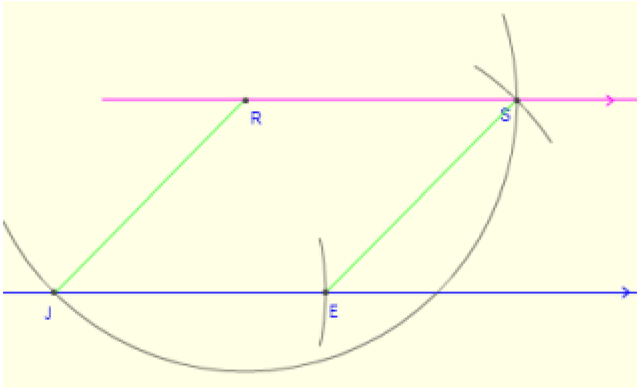
Done. The line just drawn is a perpendicular to the line at K

	Argument	Reason
1	Segment KP is congruent to KQ	They were both drawn with the same compass width
2	Segment PR is congruent to QR	They were both drawn with the same compass width
3	Triangles ΔKRP and ΔKRQ arecongruent	Three sides congruent (sss). KR is common to both.
4	Angles PKR, QKR arecongruent	CPCTC. Corresponding parts of congruent triangles are congruent
5	Angles PKR QKR are both 90°	They are a linear pair and (so add to 180°) and congruent (so each must be 90°)

Line parallel to given line, through given point.

This construction works by creating a rhombus. Since we know that the opposite sides of a rhombus are parallel, then we have created the desired parallel lines.

NOTE: There are various methods to constructing this line



Start with a line segment PQ and a point R off the line.

1. Place the compasses on point R and set its width to a little more than the distance to the line PQ.
    a. The exact distance is not important.
2. Draw a wide arc from the right of R around so it crosses the line PQ at two points.
    a. Label the left point J
3. Without adjusting the compasses' width, move the compasses to J and draw an arc across the line PQ. Label this point E.
4. Without adjusting the compasses' width, move the compasses to E and draw an arc across the large arc to the right of R. Label this point S.
5. Draw a straight line through points R and S.

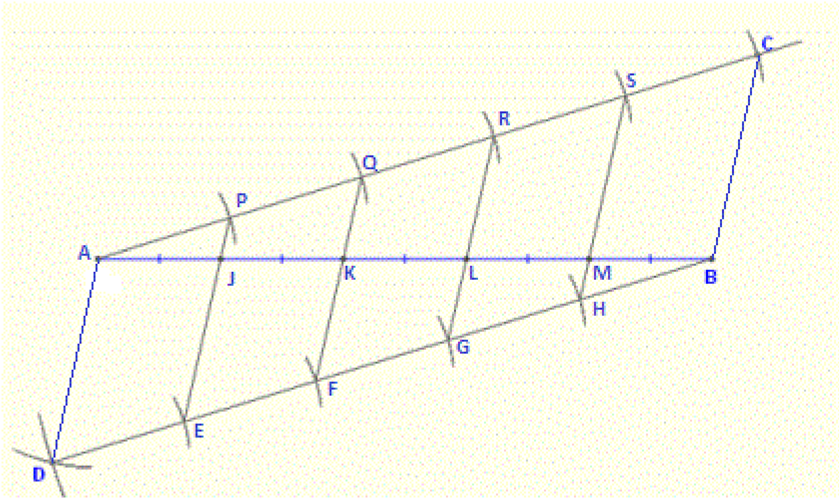
Done. The line RS is parallel to the line PQ

	Argument	**Reason**
1	Line segments RJ, JE, ES, RS arecongruent	All drawn with the same compass width.
2	RJES is a rhombus	A rhombus is a quadrilateral with 4 congruent sides.
7	Lines RS and JE are parallel	Opposite sides of a rhombus are always parallel. SeeDefinition of a Rhombus

Division of a segment into 2, 3 equal segments, without measuring it.

Or

Division of a segment into any number of equal segments, without measuring it.



Start with a line segment AB that we will divide up into 5 (in this case) equal parts.

1. From point A, draw a line segment at an angle to the given line, and about the same length. The exact length is not important.
2. Set the compasses on A, and set its width to a bit less than one fifth of the length of the new line.
3. Step the compasses along the line, marking off 5 arcs. Label the last one C.
4. With the compasses' width set to CB, draw an arc from A just below it.
5. With the compasses' width set to AC, draw an arc from B crossing the one drawn in step 4. This intersection is point D.
6. Draw a line from D to B.
7. Using the same compasses' width as used to step along AC, step the compasses from D along DB making 4 new arcs across the line
8. Draw lines between the corresponding points along AC and DB.

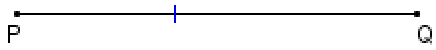
Done. The lines divide the given line segment AB in to 5 congruent parts.

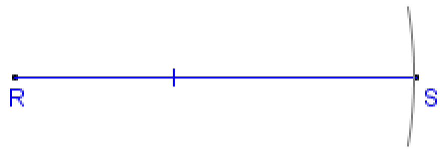
	Argument	Reason
We first prove that AC, DB are parallel		
1	AC = DB	By construction. See Copying a line segment for method and proof
2	AD = CB	By construction. Compass width for AD set from CB
3	ACBD is a parallelogram.	A quadrilateral with congruent opposite sides is aparallelogram.
4	AC, DB are parallel	Opposite sides of a parallelogram are parallel.
We next prove that PE, QF are parallel		
5	PQ = EF	Drawn with same compass width
6	PQ, EF are parallel	From (4)

	Argument	Reason
7	PQFE is a parallelogram.	A quadrilateral with one pair of opposite sides parallel and congruent is a parallelogram.
8	PE, GF are parallel	Opposite sides of a parallelogram are parallel.
Prove that triangle AQK is similar to and twice the size of APJ		
9	∠APJ = ∠AQK	Corresponding angles. AB is a transversal across the parallels PE, QF
10	∠AJP = ∠AKQ	Corresponding angles. AB is a transversal across the parallels PE, QF
11	Triangles AQK, APJ are similar	AAA. ∠PAJ is common to both, and (9), (10). See Similar triangles test, angle-angle-angle.
12	Triangles AQK is twice the size of APJ	AP = PQ. Both drawn with same compass width.
Prove that AJ = JK		
13	AK is twice AJ	(11), (12). AQK is similar to, and twice the size of APJ. All sides of similar triangles are in the same proportion. SeeProperties of similar triangles .
14	AJ = JK	From (13), J must be the midpoint of AK.
We have proved the first two segments along the given line AB are congruent. We repeat steps 5-14 for each successive triangle. For example we show that triangle ARL is similar to and three times APJ, and so AJ is one third AL. We continue until we have shown that all the segments along AB are congruent.		
15	AJ = JK = KL = LM = MB	By applying the same steps to triangle AQK, ARL etc.
16	AB is divided into n equal parts.	

Line segment of given length on a given ray.

Start with a line segment PQ that we will copy.   Geometry construction with compass and straightedge or ruler or ruler





1. Mark a point R that will be one endpoint of the new line segment.
2. Set the compasses' point on the point P of the line segment to be copied.
3. Adjust the compasses' width to the point Q. The compasses' width is now equal to the length of the line segment PQ.
4. Without changing the compasses' width, place the compasses' point on the the point R on the line you drew in step 1
5. Without changing the compasses' width, Draw an arc roughly where the other endpoint will be.
6. Pick a point S on the arc that will be the other endpoint of the new line segment.
7. Draw a line from R to S.

Done. The line segment RS is equal in length (congruent to) the line segment PQ.

	Argument	Reason
1	All points along the the arc S are the same distance from R	R is the center of the arc. See Arc definition
2	This distance is equal to the length of segment PQ	The arc was drawn with that compass width
3	RS is congruent to PQ	S is a point on the arc. See (1).

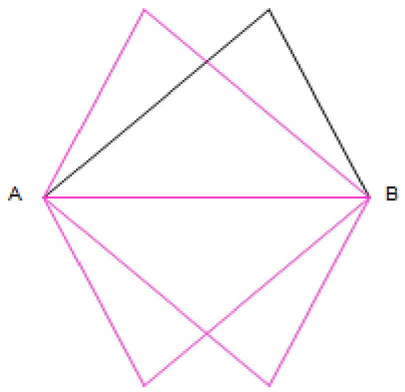
Angle of given number of degrees with a given ray as one arm.

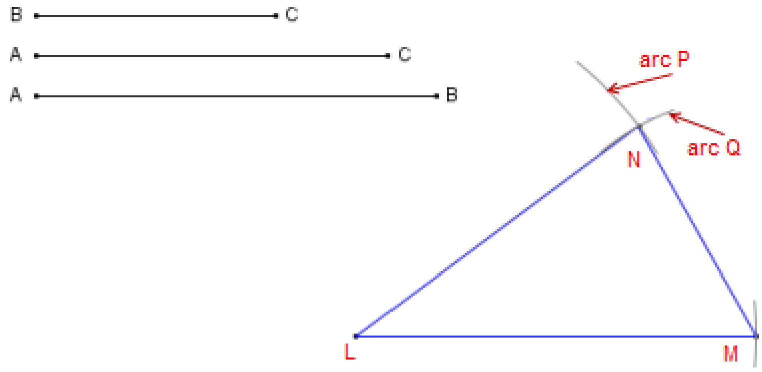
Trivial... Protractor

Triangle, given lengths of three sides.

Multiple triangles possible

It is possible to draw more than one triangle that has three sides with the given lengths. For example in the figure below, given the base AB, you can draw four triangles that meet the requirements. All four are correct in that they satisfy the requirements, and are congruent to each other.





Start with three line segments that will be the three sides of the triangle ABC.

1. Mark a point A that will be one vertex of the new triangle.
2. Set the compasses' width to the length of the segment AB. This will become the base of the new triangle.
3. With the compasses' point on A, make an arc near the future vertex B of the triangle.
4. Mark a point B on this arc. This will become the next vertex of the new triangle.
5. Set the compasses' width to the length of the line segment AC.
6. Place the compasses' point on A and make an arc in the vicinity of where the third vertex of the triangle (C) will be. All points along this arc are the distance AC from A, but we do not yet quite know exactly where the vertex C is.
7. Use the compasses to measure the length of the segment BC, the length of the third side of the triangle.
8. From point B, draw an arc crossing the first. Where these intersect is the vertex C of the triangle
9. Finally, draw the three sides AB, AC, and BC of the new triangle.

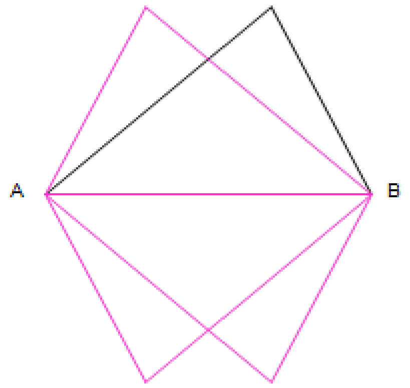
Done. The blue triangle ABC has each side congruent to the corresponding line segment.

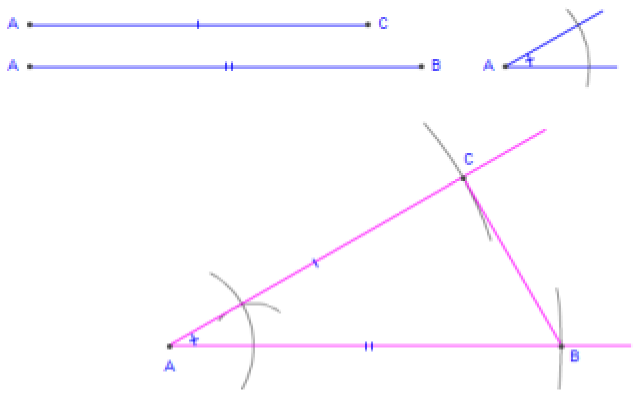
	Argument	Reason
1	Line segment LM is congruent to AB.	Drawn with the same compass width. See Copying a line segment
2	The third vertex N of the triangle must lie somewhere on arc P.	All points on arc P are distance AC from L since the arc was drawn with the compass width set to AC.
3	The third vertex N of the triangle must lie somewhere on arc Q.	All points on arc Q are distance BC from M since the arc was drawn with the compass width set to BC.
4	The third vertex N must lie where the two arcs intersect	Only point that satisfies 2 and 3.
5	Triangle LMN satisfies the three side lengths given. LM is congruent to AB, LN is congruent to AC, MN is congruent to BC,	

Triangle, given SAS data.

Multiple triangles possible

It is possible to draw more than one triangle has the side lengths and angle measure as given. Depending on which line you start with, which end of the line you draw the angles, and whether they are above or below the line, the four triangles below are possible. All four are correct in that they satisfy the requirements, and are congruent to each other.





Start with two line segments and the included angle.

1. Mark a point A that will be one vertex of the new triangle.
2. Draw a ray from point A. This will become the side AB of the new triangle, so make it longer than AB.
3. Set the compasses' width to the length of the given side AB.
4. Set the compasses on A, and mark a point B on the ray just drawn.
5. Set the compasses' width to the distance AC.
6. Using a protractor find a point corresponding to the given angle
7. Construct a ray from A to this point
8. With the compasses on A, make an arc across the second ray, creating point C.
9. Draw the line BC, the third side of the triangle

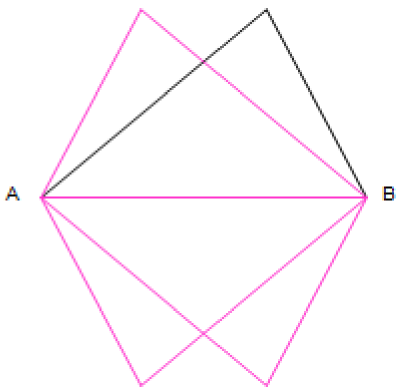
Done, the triangle ABC has the desired two side lengths and included angle.

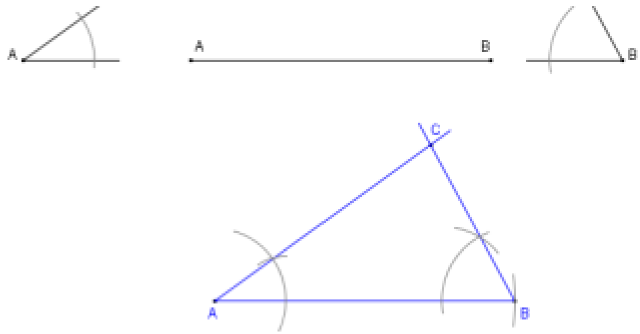
	Argument	Reason
1	Line segment MN is congruent to AB.	Drawn with the same compass width. For proof seeCopying a line segment
2	Line segment ML is congruent to AC.	Drawn with the same compass width.
3	The angle LMN is congruent to the angle A	Copied using the procedure shown in Copying an angle. See that page for the proof.
4	Triangle LNM satisfies the side lengths and angle measure given.	

Triangle, given ASA data.

Multiple triangles possible

It is possible to draw more than one triangle has the side length and angle measures as given. Depending on which end of the line you draw the angles, and whether they are above or below the line, four triangles are possible. All four are correct in that they satisfy the requirements, and are congruent to each other.





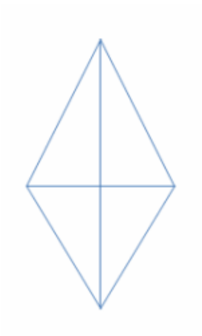
1. Mark a point A that will be one vertex of the new triangle.
2. Draw a ray from point A. This will become the side AB of the new triangle, so make it longer than AB.
3. Set the compasses' width to the length of the given side AB.
4. Set the compasses on A, and mark a point B on the ray just drawn.
5. Using a protractor construct the angles to measure given
6. Draw rays from A to the given angle, repeat from B
7. Draw the line AC, the second side of the triangle
8. Draw the line BC, the third side of the triangle

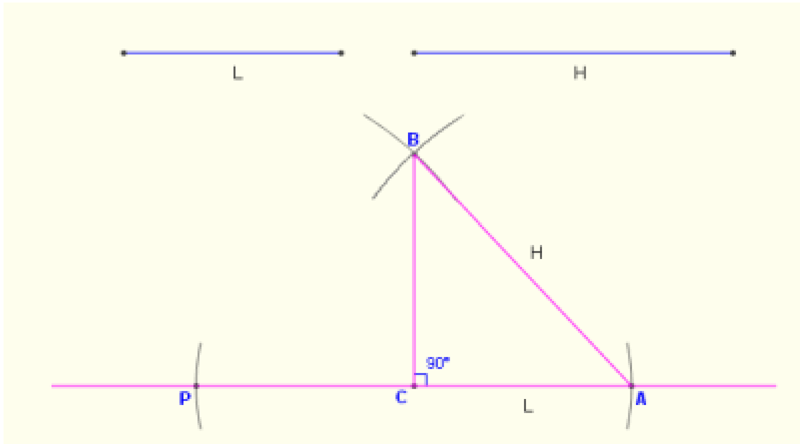
Done, the triangle ABC has the desired two side lengths and included angle.

	Argument	Reason
1	Line segment JL is congruent to AB.	Drawn with the same compass width. For proof seeCopying a line segment
2	The angle KJL is congruent to the angle A	Copied using the procedure shown in Copying an angle. See that page for the proof.
3	The angle KLJ is congruent to the angle B	Copied using the procedure shown in Copying an angle. See that page for the proof.
4	Triangle JKL satisfies the side length and two angle measure given.	

Right-angled triangle, given the length of the hypotenuse and one other side.

Multiple triangles possible. It is possible to draw more than one triangle has the side lengths as given. Youcan use the triangle to the left or right of the initial perpendicular, and also draw them below the initial line. All four are correct in that they satisfy the requirements, and arecongruent to each other.





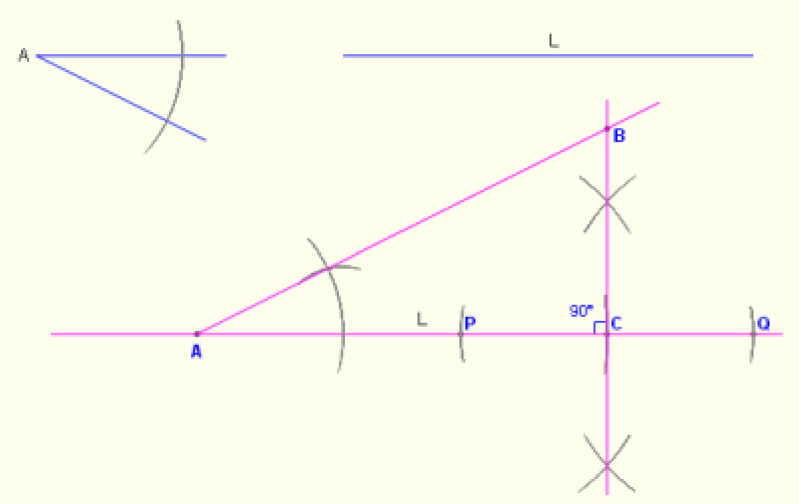
Start with the given segment lengths for the hypotenuse (H) and the given leg (L).

1 Draw a long horizontal line
2 Mark a point C somewhere near the middle of that line.
3 Set the compass width to the length of the given leg L
4 With the compasses on C, mark an arc on each side of C, creating points P and A
5 Set the compass' width to the length of the given hypotenuse H.
6 With the compasses on P, make an arc above C
7 Without changing the compass width, move to A and make another arc over C, creating point B.
8 Using the straight edge, draw the lines BC and BA

Done. ABC is a right triangle with the given leg and hypotenuse length

	Argument	Reason
We first prove that ΔBCA is a right triangle		
1	CP is congruent to CA	They were both drawn with the same compass width
2	BP is congruent to BA	They were both drawn with the same compass width
3	CB is common to both triangles BCP and BCA	Common side
4	Triangles ΔBCP and ΔBCA arecongruent	Three sides congruent (SSS).
5	∠BCP, ∠BCA are congruent	CPCTC. Corresponding parts of congruent triangles are congruent
6	m∠BCA = 90°	∠BCA and ∠BCP are a linear pair and (so add to 180°) andcongruent so each must be 90°
7	CA is congruent to the given leg L	CA copied from L. See Copying a segment.
8	AB is congruent to the given hypotenuse H	Drawn with same compass width
9	ΔBCA is a right triangle with the desired side lengths	From (6), (7), (8)

Right-angled triangle, given one side and one of the acute angles (several cases).

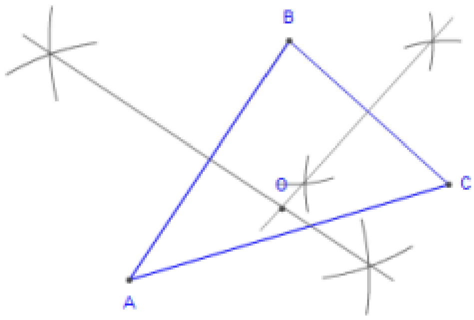


1. Copy the angle A. (See Copying an angle)or construct an angle using a protractor
2. Copy the length of the given leg onto the bottom angle leg (See Copying a segment)
3. Erect a perpendicular from the end of the leg. (See Perpendicular to a line at a point)

	Argument	Reason
We first prove that ∆BCA is a right triangle		
1	m∠BCA = 90°	BC was constructed using the procedure inPerpendicular to a line at a point. See that page for proof.
2	Therefore ∆BCA is a right triangle	By definition of a right triangle, one angle must be 90°
Now prove AC is congruent to the given leg		
3	AC = the given leg	AC was copied from the leg at the same compass width
Now prove ∠BAC is the given angle A		
4	m∠BAC = given m∠A	Copied using the procedure in Copying an angle. See that page for proof
9	∆BCA is a right triangle with the desired hypotenuse H and angle A	From (2), (3), (4)

Circumcentre and circumcircle of a given triangle, using only straightedge and compass.

The circumcenter of a triangle is the point where the perpendicular bisectors of the sides intersect. It is also the center of the circumcircle, the circle that passes through all three vertices of the triangle. This construction assumes you are already familiar with Constructing the Perpendicular Bisector of a Line Segment.



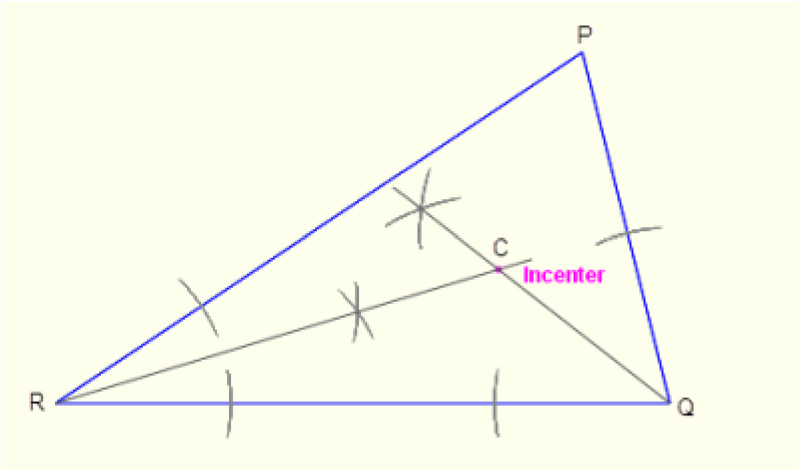
1. Find the bisector of one of the triangle sides. Any one will do.
2. Repeat for another side. Any one will do.
3. Mark the point where these two perpendiculars intersect as point O.

(Optional step) Repeat for the third side. This will convince you that the three bisectors do, in fact, intersect at a single point. But two are enough to find that point.
The point O is the circumcenter of the triangle ABC.

Incentre and incircle of a given triangle, using only straight-edge and compass.

The Incenter of a triangle is the point where all three angle bisectors always intersect, and is the center of the triangle's incircle.

In this construction, we only use two bisectors, as this is sufficient to define the point where they intersect, and we bisect the angles using the method described in Bisecting an Angle. The point where the bisectors cross is the incenter.

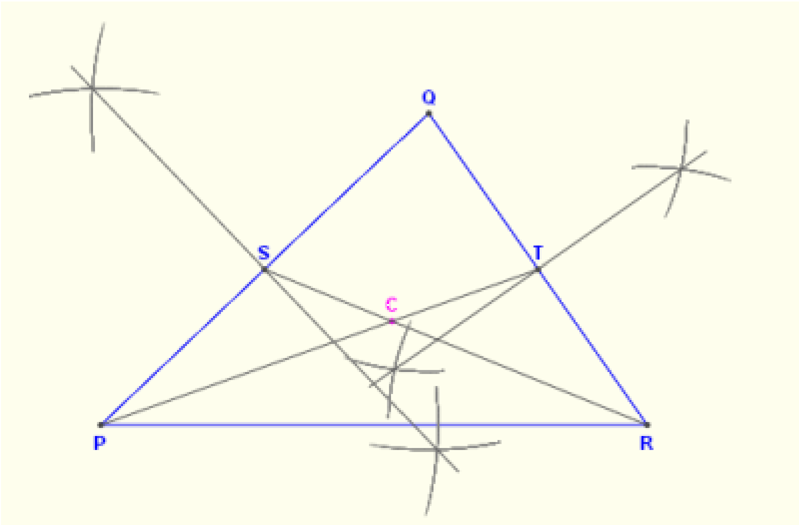


1. Place the compasses' point on any of the triangle's vertices. Adjust the compasses to a medium width setting. The exact width is not important.
2. Without changing the compasses' width, strike an arc across each adjacent side.
3. Change the compasses' width if desired, then from the point where each arc crosses the side, draw two arcs inside the triangle so that they cross each other, using the same compasses' width for each.
4. Using the straightedge, draw a line from the vertex of the triangle to where the last two arcs cross.
5. Repeat all of the above at any other vertex of the triangle. You will now have two new lines drawn.
6. Done. Mark a point where the two new lines intersect. This is the incenter of the triangle.

(Optional) Repeat steps 1-4 for the third vertex. This will convince you that the three angle bisectors do, in fact, always intersect at a single point. But two are enough to find that point.

Centroid of a triangle.

The centroid of a triangle is the point where its medians intersect. It works by constructing the perpendicular bisectors of any two sides to find their midpoints. Then the medians are drawn, which intersect at the centroid. This construction assumes you are already familiar with Constructing the Perpendicular Bisector of a Line Segment.

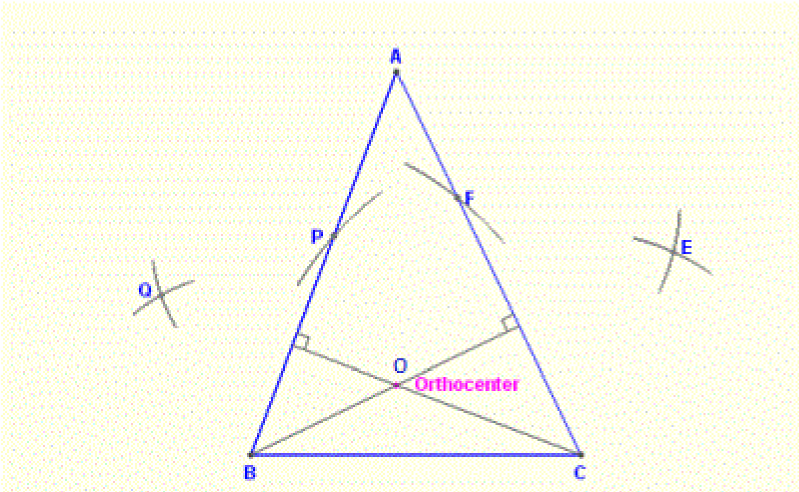


1. Construct the bisector of the line segment PQ. Label the midpoint of the line S.
2. Draw the median from the midpoint S to the opposite vertex R. Next, we draw the second median of the triangle through P
3. In the same manner, construct T, the midpoint of the line segment QR.
4. Draw the median from the midpoint T to the opposite vertex P

(Optional step) Repeat for the third side. This will convince you that the three medians do in fact intersect at a single point. But two are enough to find that point.
Done. The point C where the two medians intersect is the centroid of the triangle PQR.

Orthocentre of a triangle

The orthocenter is the point where all three altitudes of the triangle intersect. An altitude is a line which passes through a vertex of the triangle and is perpendicular to the opposite side. There are therefore three altitudes in a triangle. It works using the construction for a perpendicular through a point to draw two of the altitudes, thus location the orthocenter.



1. Set the compasses' width to the length of a side of the triangle. Any side will do, but the shortest works best.
2. With the compasses on B, one end of that line, draw an arc across the opposite side. Label this point F
3. Repeat for the other end of the line, C. Label this point P.

Note If you find you cannot draw these arcs on the opposite sides, the orthocenter is outside the triangle.

4. With the compasses on B, set the compasses' width to more than half the distance to P.
5. From B and P, draw two arcs that intersect, creating point Q.
6. Use a straightedge to draw a line from C to Q. The part of this line inside the triangle forms an altitude of the triangle.

Now we repeat the process to create a second altitude.

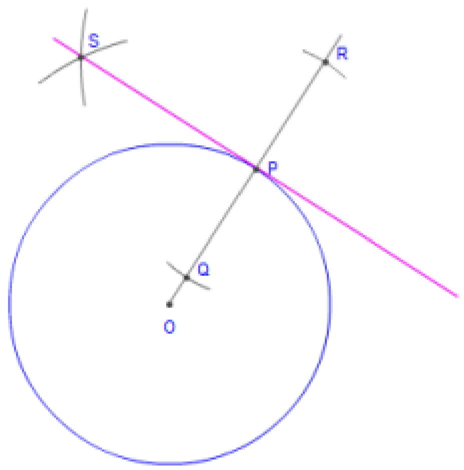
7. With the compasses on C, set the compasses' width tomore than half the distance to F.
8. From C and F, draw two arcs that intersect, creating point E.
9. Use a straightedge to draw a line from B to E. The part of this line inside the triangle forms an altitude of the triangle.
10. Done. The point where the two altitudes intersect is the orthocenter of the triangle. (You may need to extend the altitude lines so they intersect if the orthocenter is outside the triangle)

Optional Step 11.

Repeat steps 7,8,9 on the third side of the triangle. This will help convince you that all three altitudes do in fact intersect at a single point. But two altitudes are enough to find that point.

Angle of 60°, without using a protractor or set square.

Tangent to a given circle at a given point on it.



1. Draw a straight line from the center O, through the given point P and on beyond P.
2. Put the compasses' point on P and set it to any width less than the distance OP. Then, on the line just drawn, draw an arc on each side of P. This creates the points Q and R as shown.
3. Set the compasses on Q and set it to any width greater than the distance QP.
4. Without changing the compasses' width, draw an arc approximately in the position shown on one side of P.
5. Without changing the compasses' width, move the compasses to R and make another arc across the first, creating point S.
6. Draw a line through P and S.

Done. The line PS just drawn is the tangent to the circle O through point P.

Parallelogram, given the length of the sides and the measure of the angles.