

LC OL - Co-Ordinate Geometry of The Line

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Learning Outcomes:

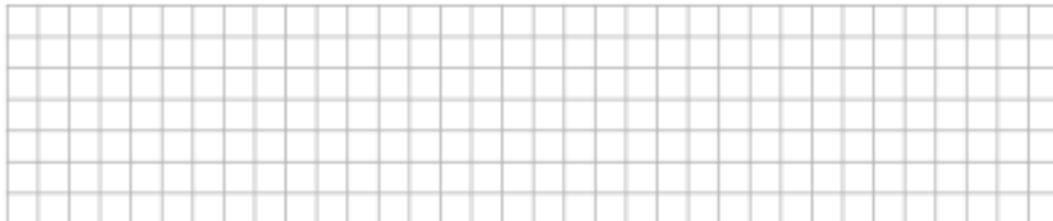
At the end of this section you should be able to do the following:

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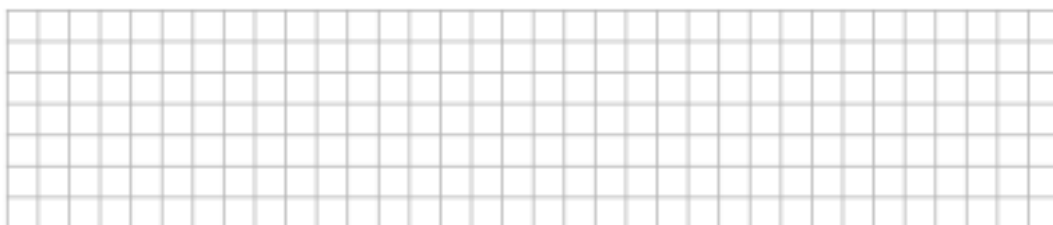
Distance between two points

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

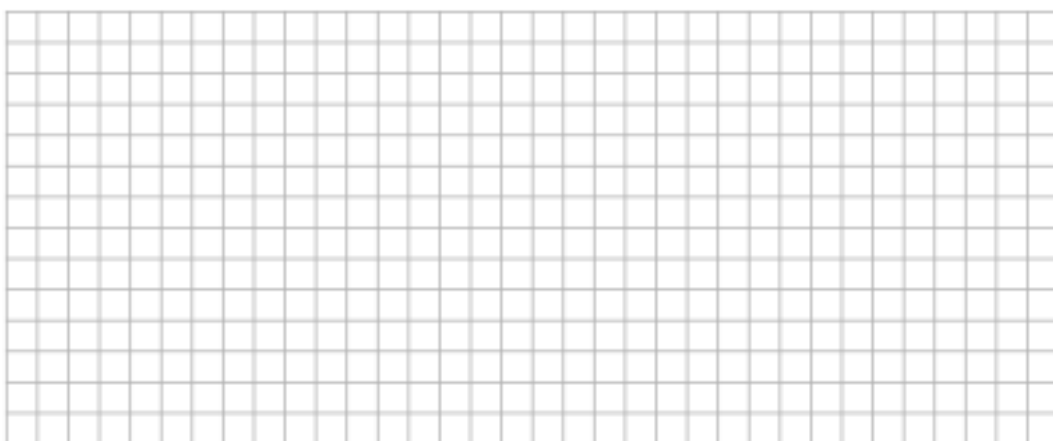
1. Find the distance between the points (-7,-3) and (-2,2)



2. a(3,6) and b(-1,3) are two points, Find $|ab|$.



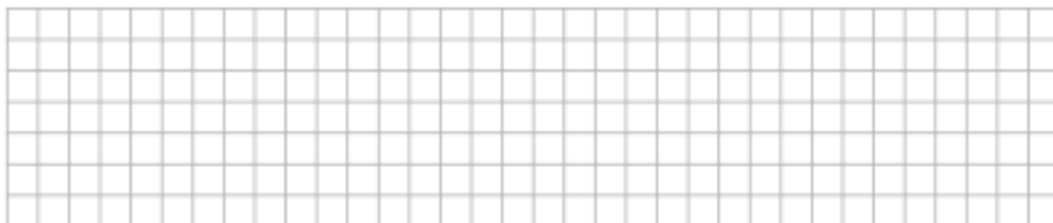
3. X(-3,1) and Y(4,-2) are two points. Find the length of the line segment $[xy]$. Give your answer in surd form.



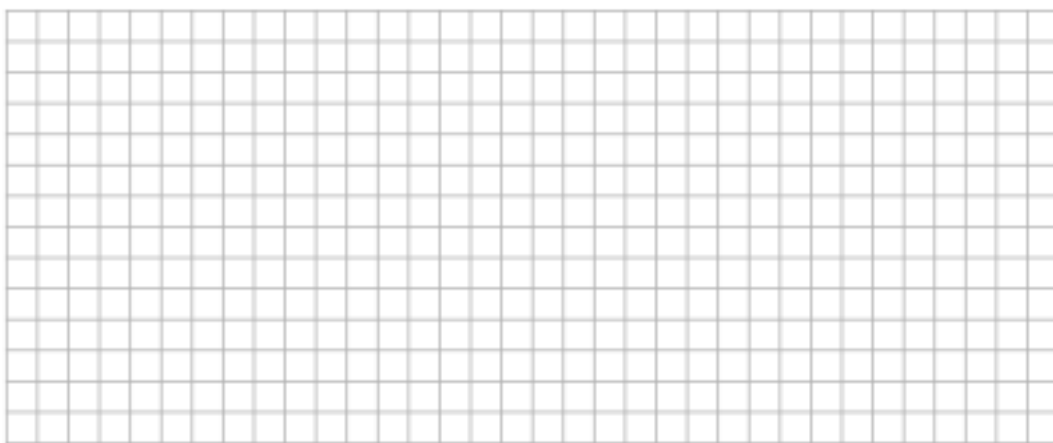
Midpoint of a line

$$\text{Midpoint} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} \right)$$

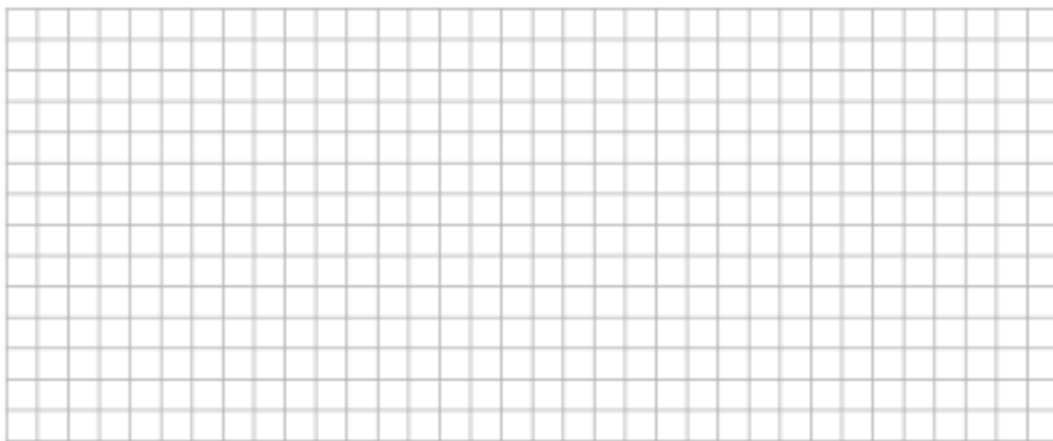
1. Find the midpoint of the line segment joining the points (-5,3) and (2,-2).



2. a(3,-2) and b(-1,1) are two points.
a. Find the co-ordinates of the midpoint of [ab].
b. Find |ab|



3. p(2,4) and q(5,1) are two points. q is the midpoint of [pr]. Find the co-ordinates of r.

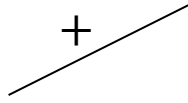


4. $s(-1,2)$ is the midpoint of $[PQ]$ and P is the point $(-2,-4)$. Find the co-ordinants of Q.

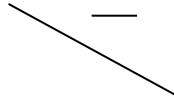


Slope of a line

- A line going up from left to right has a positive slope:



- A line going down from left to right has a negative slope:



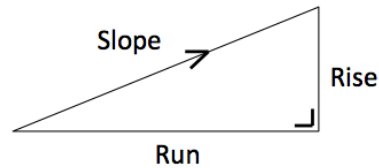
- A line going flat from left to right has a slope of zero.

Note: The greater the number the greater the slope. For example a line with a slope of 7 is steeper than a line with a slope of 5. A line with a slope of -7 would be steeper than a line with a slope of -5 (both negative so both downhill).

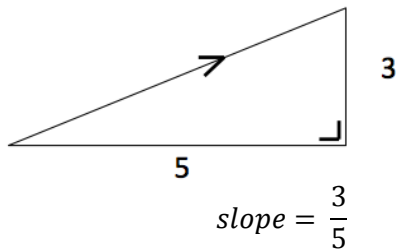
There are 4 ways to find the slope of a line:

Method 1

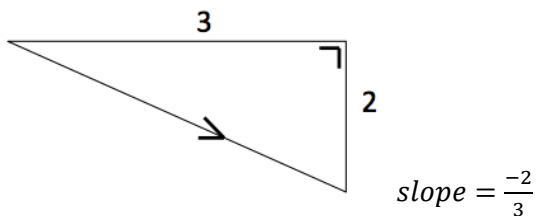
$$\text{slope} = \frac{\text{rise}}{\text{run}}$$



Eg1:



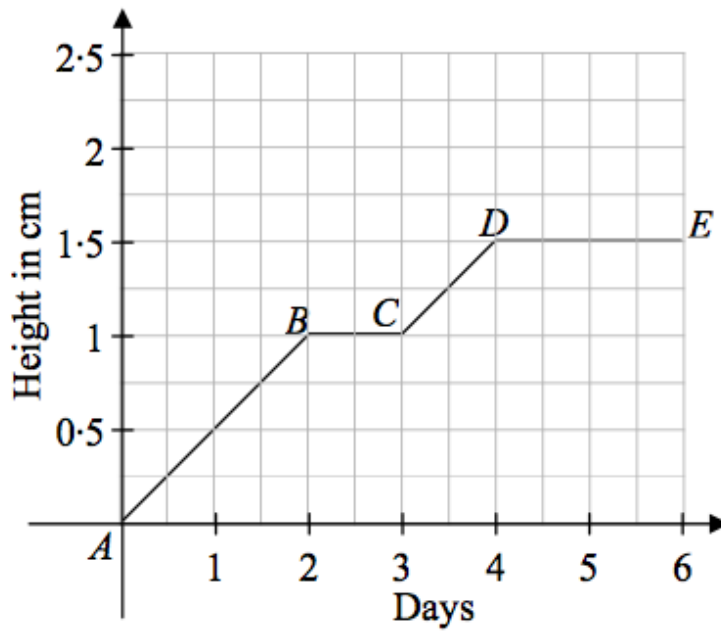
Eg2:



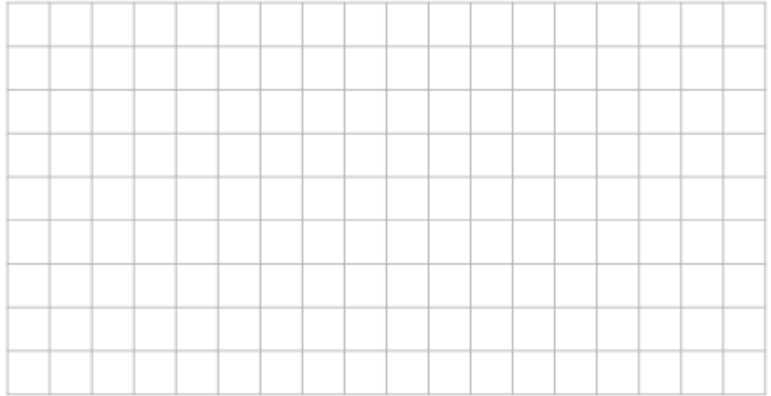
Notice that the slope is negative because it is going downhill!

1.

The height of a watercress seedling over six days is shown in the diagram below.



Find the slope of $AB =$



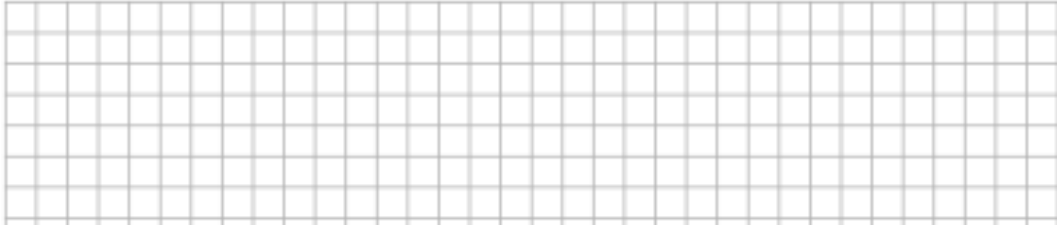
Find the slope of $CD =$

Method 2

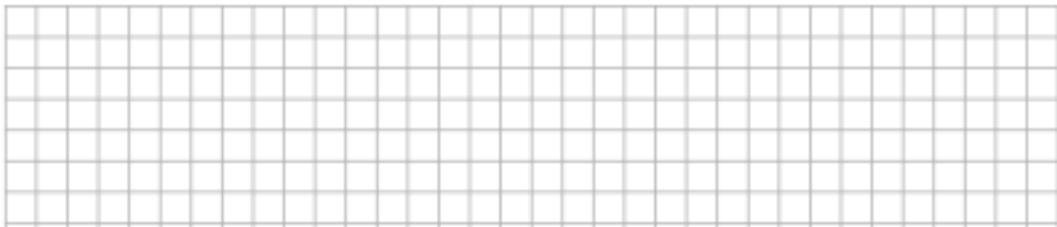
If given 2 points we use the formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

2. Find the slope of the line containing the points (2,4) and (5,9)



3. p(-1,2) and r(3,4) are two points. Find the slope of pr.

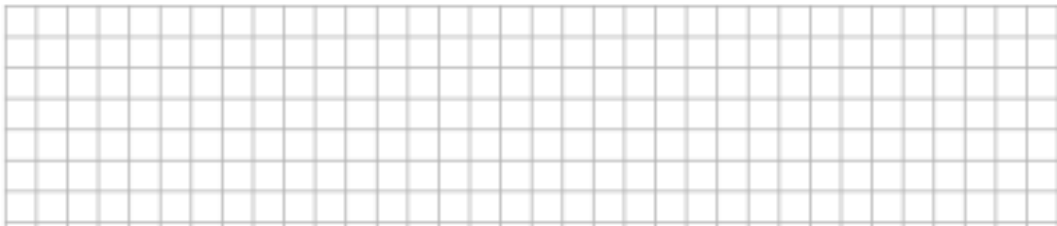


Method 3

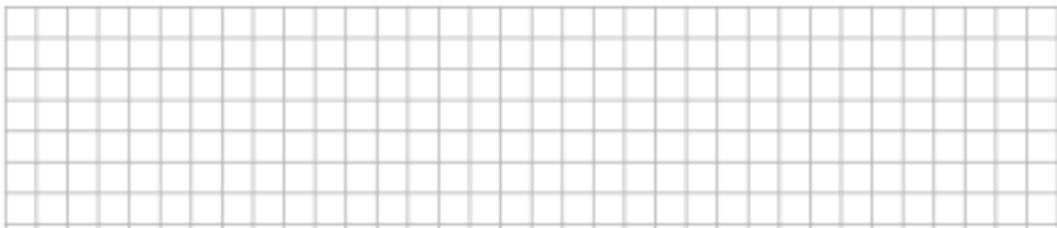
If given the equation of a line in the form $ax + by + c = 0$

$$\rightarrow \text{slope} = \frac{-a}{b}$$

4. What is the slope of the line with equation $5x + 4y - 7 = 0$



5. L is the line $2x - y + 6 = 0$. Find the slope of L.

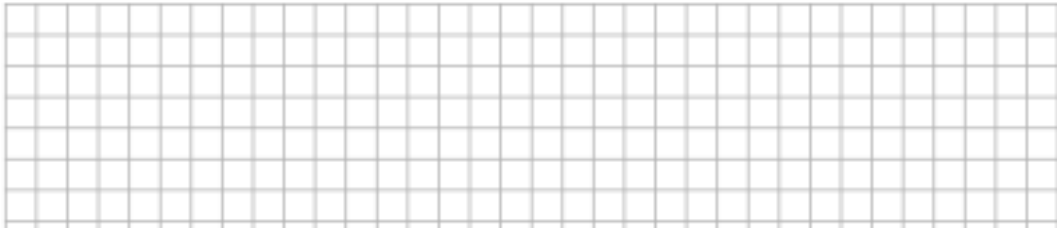


Method 4

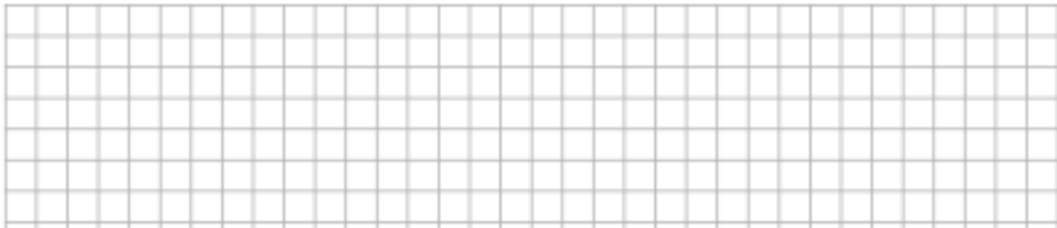
If given the equation of a line in the form $y = mx + c$.

---> m represents the slope

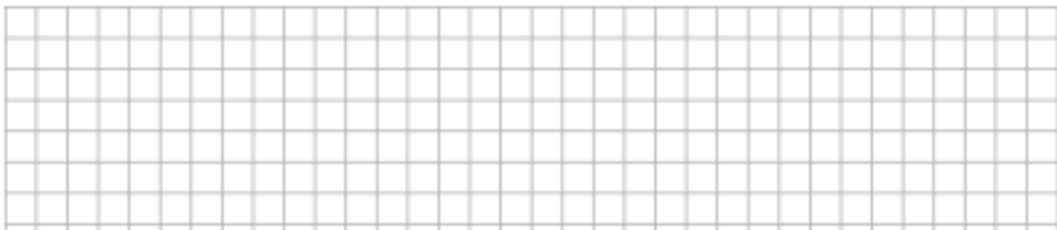
6. Find the slope of the line $y = 4x - 3$.



7. Find the slope of the line $y = -7x - 9$.



8. Find the slope of the line $y = \frac{-4x}{7} + 3$

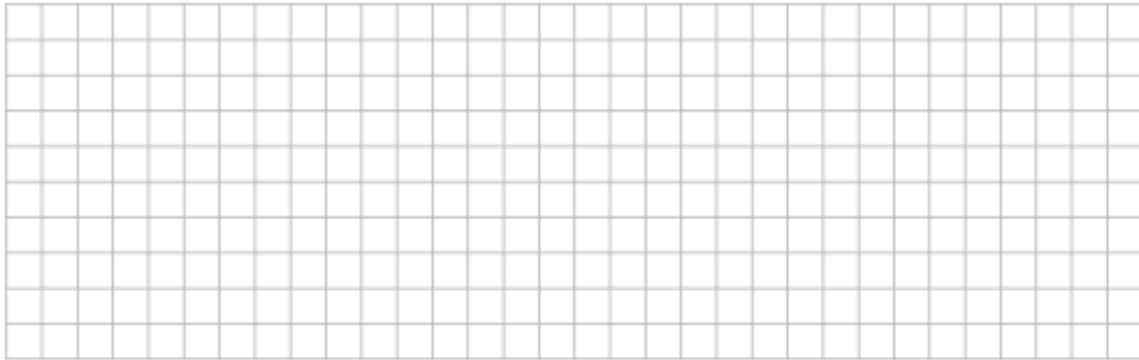


9.

The table below gives the equations of six lines.

Line 1	$y = 3x - 6$
Line 2	$y = 3x + 12$
Line 3	$y = 5x + 20$
Line 4	$y = x - 7$
Line 5	$y = -2x + 4$
Line 6	$y = 4x - 16$

Which line has the greatest slope? Give a reason for your answer.



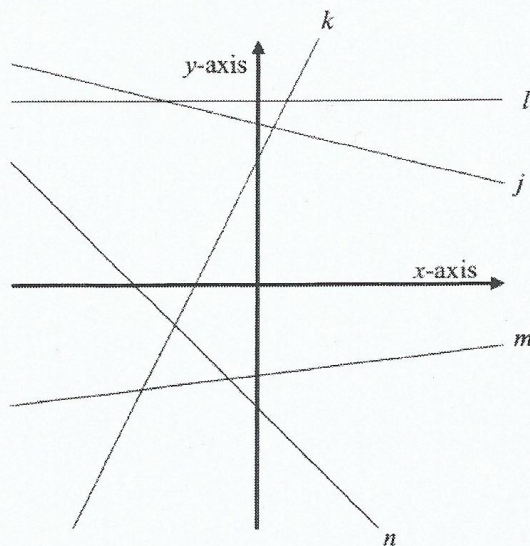
10.

(a) Five lines j , k , l , m , and n in the co-ordinate plane are shown in the diagram.

The slopes of the five lines are in the table below.

Complete the table, matching the lines to their slopes.

slope	line
2	
$\frac{1}{8}$	
0	
$-\frac{1}{4}$	
-1	



11. Write the equation of the line $5x - 2y - 12 = 0$ in the form $y = mx + c$. Hence, find its slope.



Slopes of parallel lines

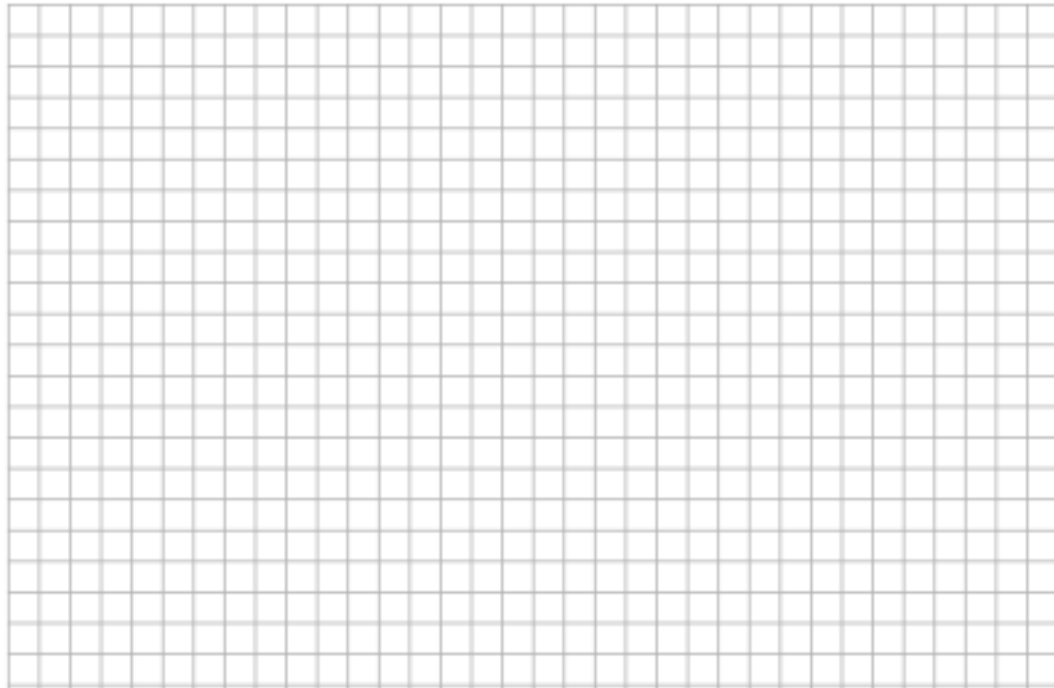
Parallel lines have equal slopes

$$(m_1 = m_2)$$

1. The equation of the line L is given by $3x - 2y + 7 = 0$, another line, N contains the points $(0,1)$ and $(4,7)$. Investigate if L and N are parallel.



2. $a(-3,0)$ $b(8,10)$ $c(-2, -2)$ $d(10,6)$ are four points. A student claims that the line from b to d is parallel to the line from a to c . Is the student correct? Give reason for your answer?



Slopes of perpendicular lines

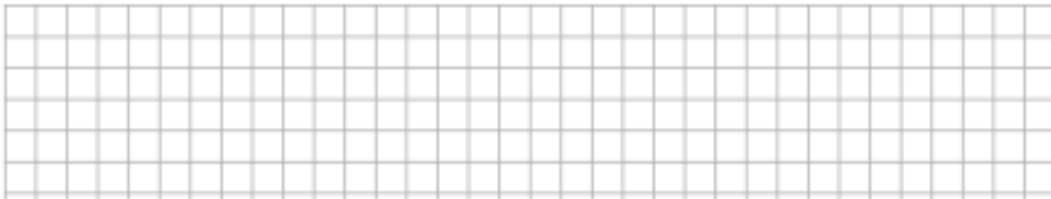
To find the slope of a perpendicular line you turn the slope upside down and change the sign

$$\text{eg1. } \frac{5}{3} \rightarrow -\frac{3}{5}$$

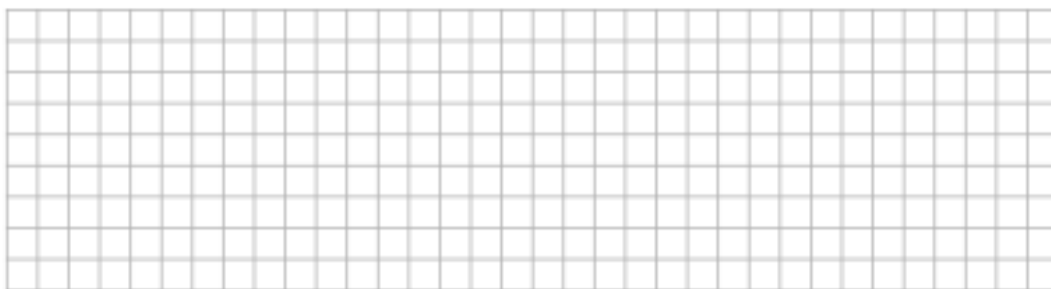
$$\text{eg2. } 7 \rightarrow -\frac{1}{7}$$

$$(m_1 \times m_2 = -1)$$

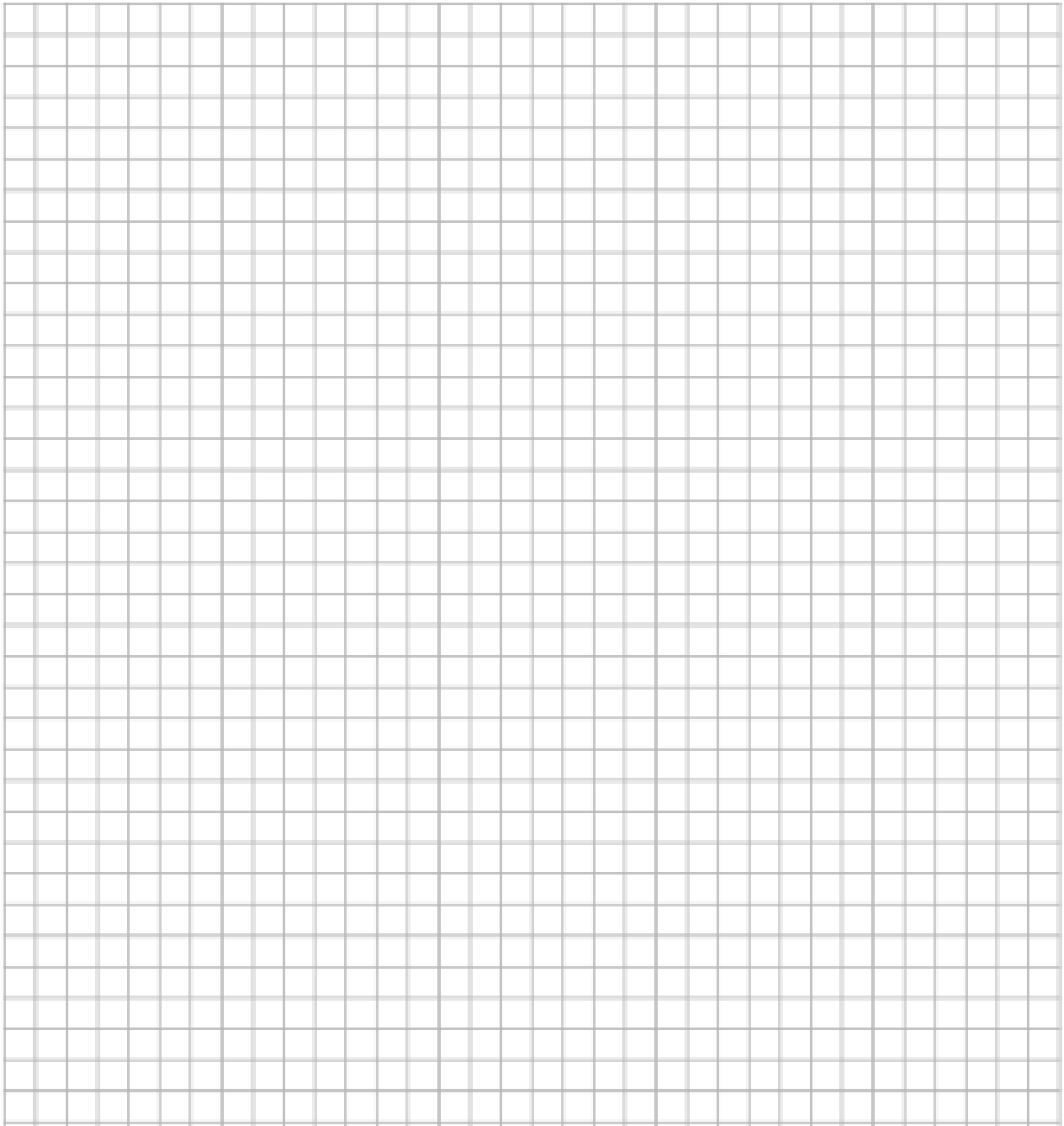
1. Show that the line segment $[pq]$ is perpendicular to the line segment $[rs]$ if the points are as follows: $p(3,4)$, $q(5,7)$, $r(-1,1)$ and $s(-4,3)$



2. L is the line $3x - y - 11 = 0$.
- Find the slope of L .
 - The line K contains the points $a(-3,0)$ and $b(6,r)$. K is perpendicular to L . Find the value of r .



3. If $p(2,3)$, $q(5,-1)$ and $r(9,2)$ are 3 points. Prove $\angle pqr$ is a right angle.



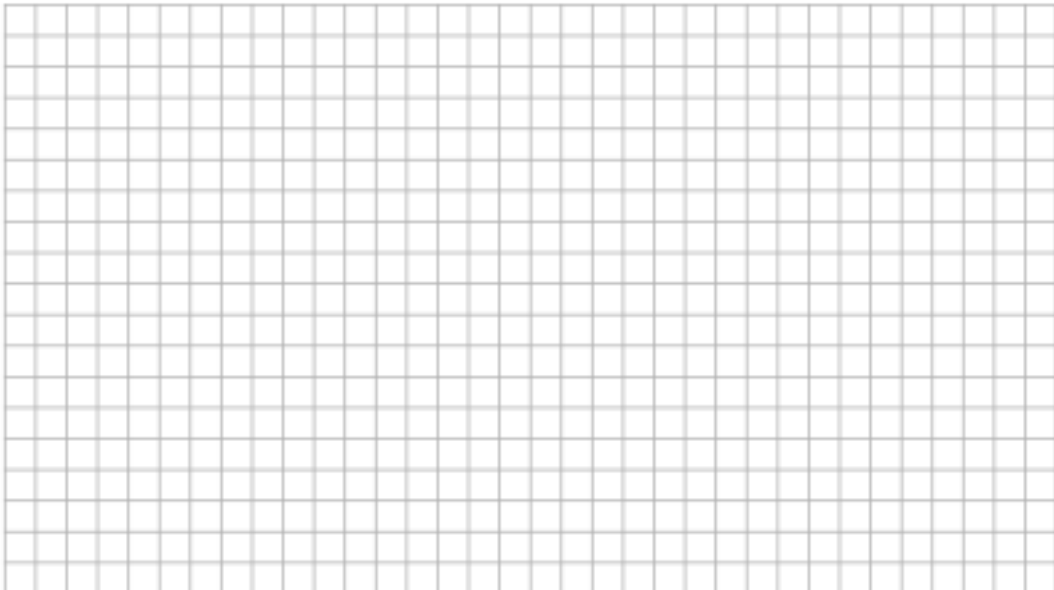
The equation of a line

$$y - y_1 = m(x - x_1)$$

1. Find the equation of the line containing the points (1,3) and (2,7).



2. $p(2,4)$ and $q(5,2)$ are two points. Find the equation of pr .



Show a Given Point is on a Line

Simply sub the point in and see if it works out equal.

1. Check if the points (2, -1) and (9, 2) are on the line $2x - 5y - 9 = 0$



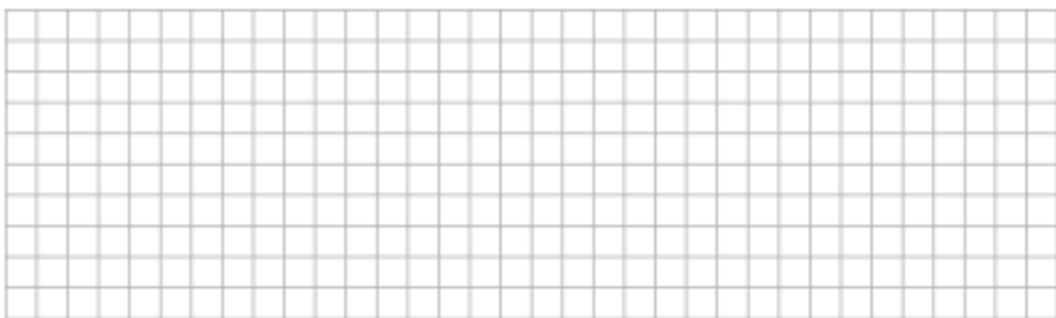
2. L is the line $2x - 3y + 5 = 0$. Show that L contains the point $p(-4, -1)$



3. Does the line $2x - y = 4$ goes through the point (2,4)?



4. K is the line $-2x + 3y + 4 = 0$. n is the point (2,0). Verify that K goes through the point n .



The point of intersection of two lines

To find the point of intersection of two lines we use simultaneous equations.

1. Find the point of intersection of the lines A and B if their equations are as follows:

A: $4x - 3y = -25$

B: $3x + 5y = 3$



2. L is the line $3x - 4y + 7 = 0$

M is the line $4x + 3y - 24 = 0$

L and M intersect at the point r . Find the co-ordinates of r .

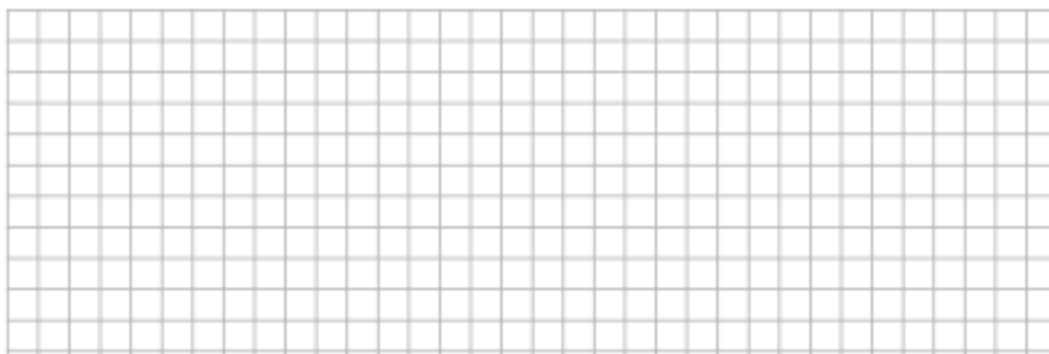


Where lines cut the x-axis and y-axis

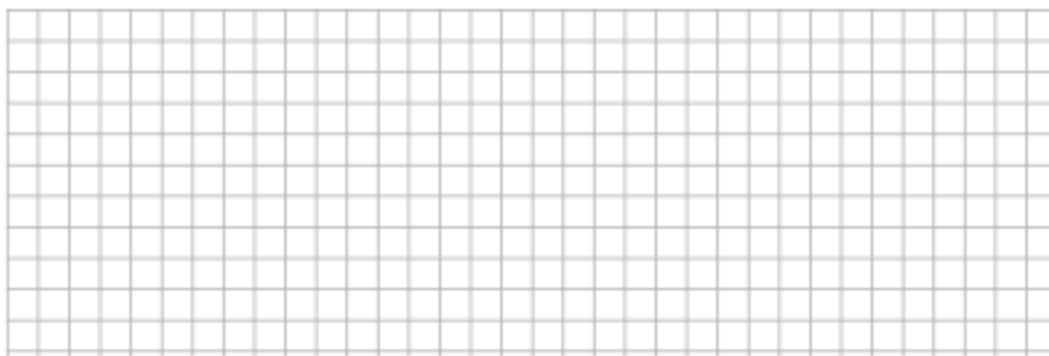
On the x-axis: $y=0$

On the y-axis: $x=0$

1. Find where the line $2x + 3y - 12 = 0$ cuts the x-axis and y-axis.



2. The line $2x - 3y + 9 = 0$ cuts the x-axis at p and the y-axis at q . Find the co-ordinates of p and the co-ordinates of q .



Graphing lines

To graph a line all you need is two points on the line.

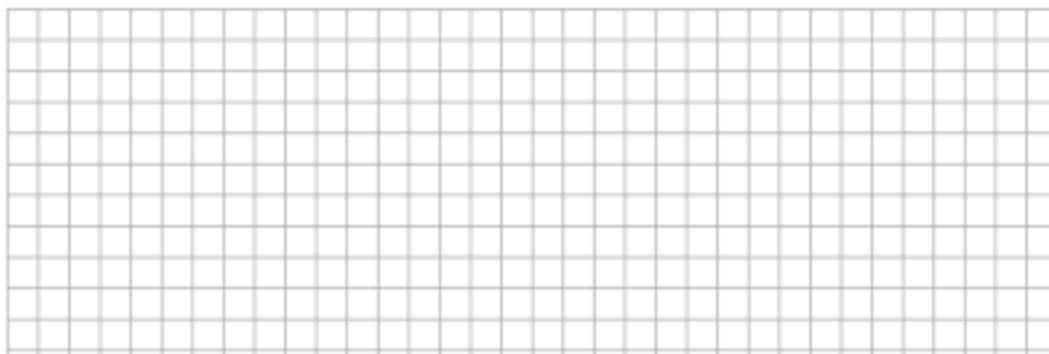
- Let $x = 0$ and find what you get for y .
- Then let $y = 0$ and find what you get for x

Once you have the two points, plot them on a graph and join the points together with a line.

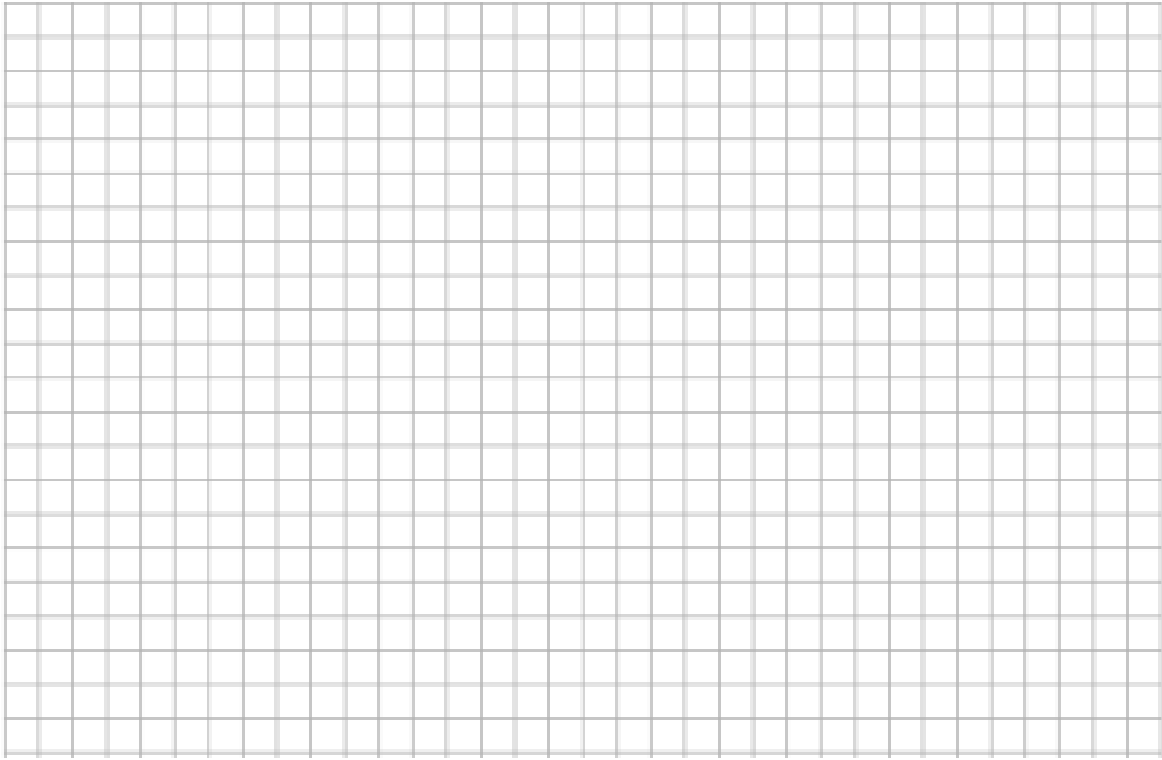
1. Graph the line $2x + y + 6 = 0$



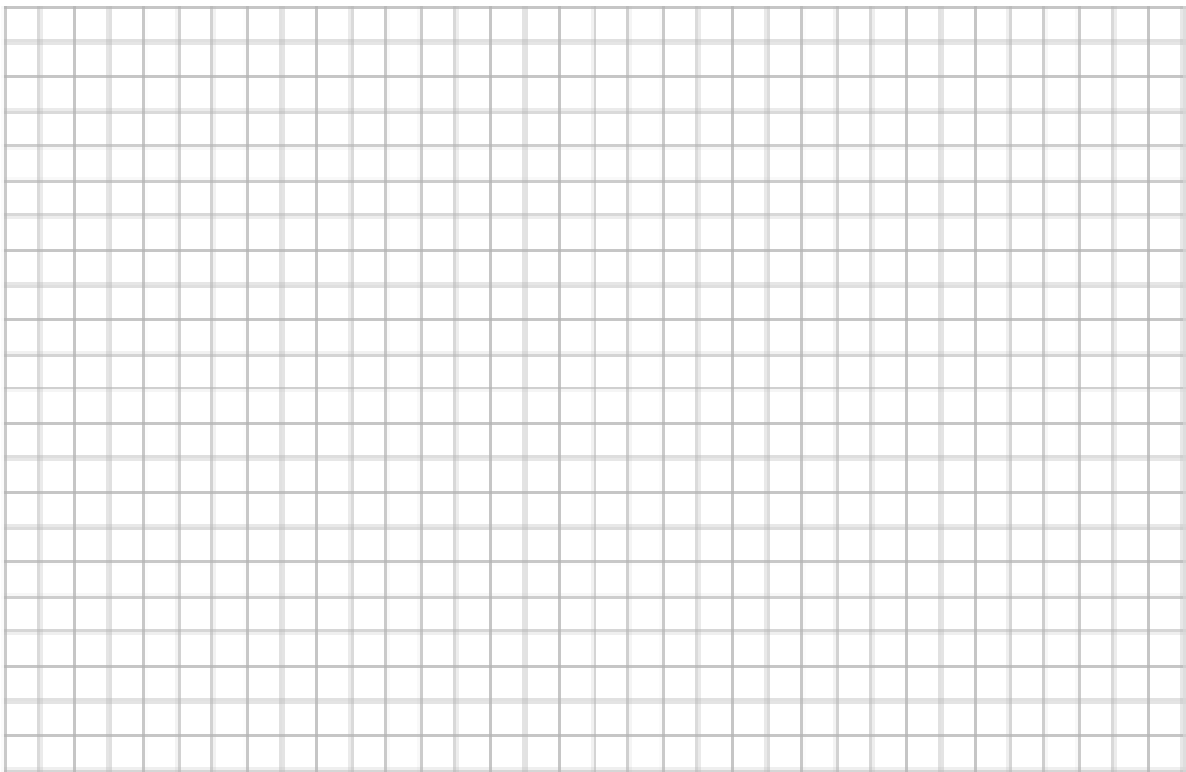
2. L is the line $3x - 2y + 12 = 0$. Show L on a co-ordinant diagram.



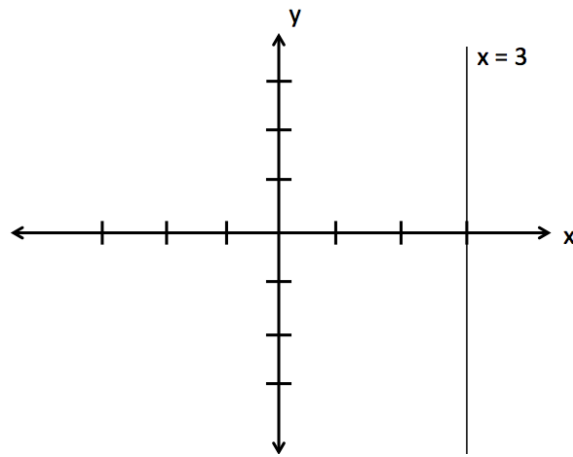
3. Sketch the line $y = 2x + 4$



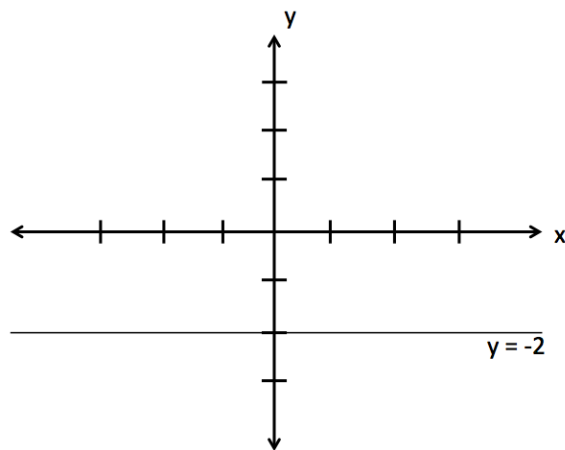
4. K is the line $2y = 8x - 2$. Show K on a co-ordinate diagram.



Draw the line $x = 3$

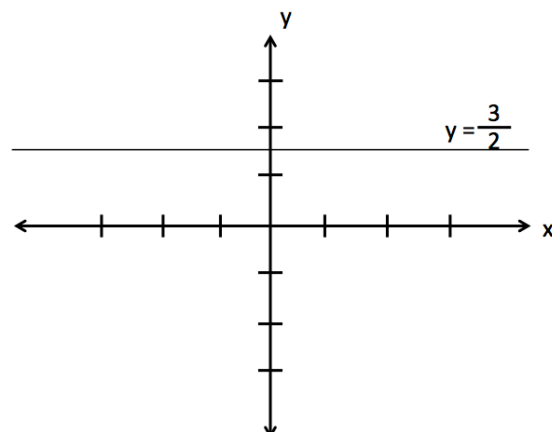


Draw the line $y = -2$

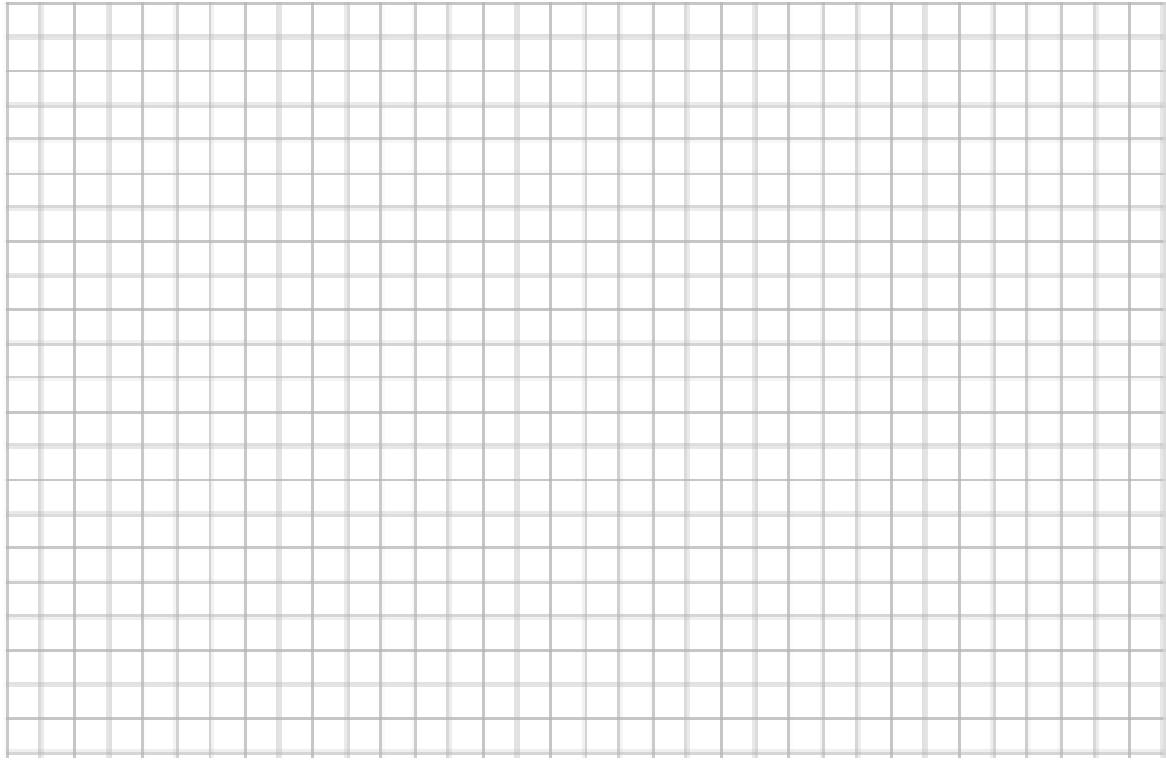


Draw the line $2y = 3$

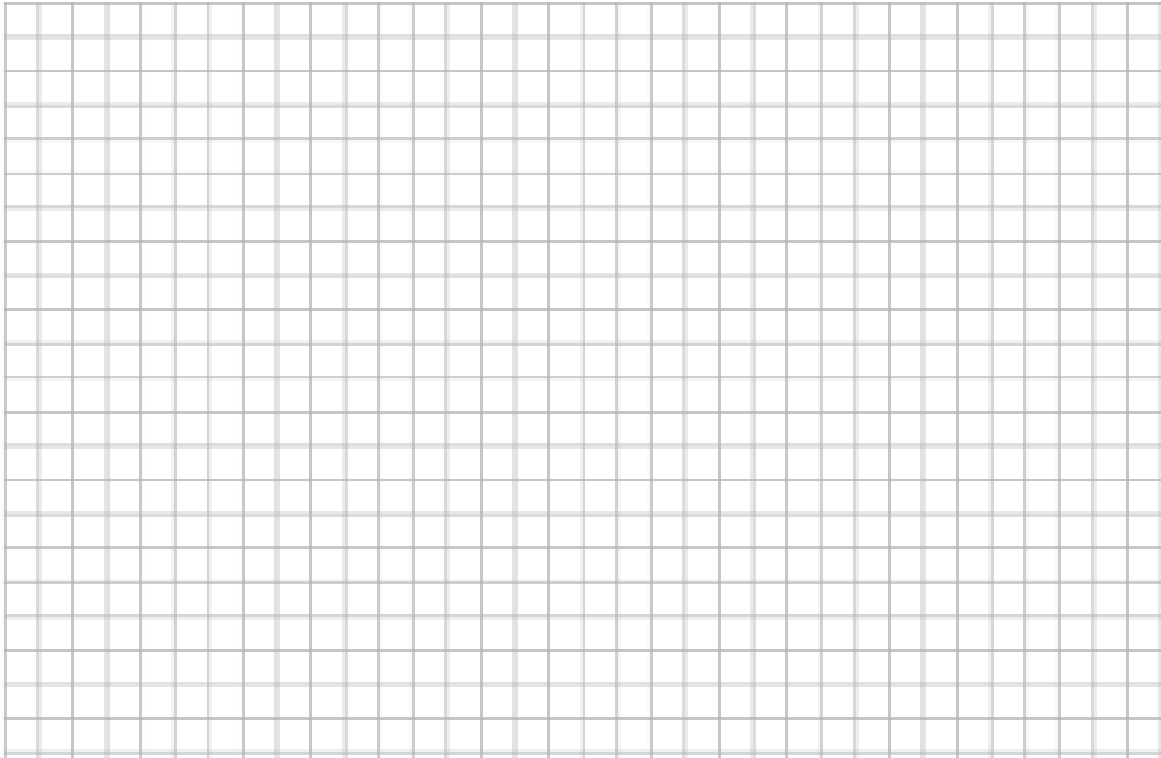
$$y = \frac{3}{2}$$
$$= 1.5$$



5. Sketch the line $x = -3$



6. The line K is given by the equation $2y = 7$. Show K on a co-ordinate diagram.



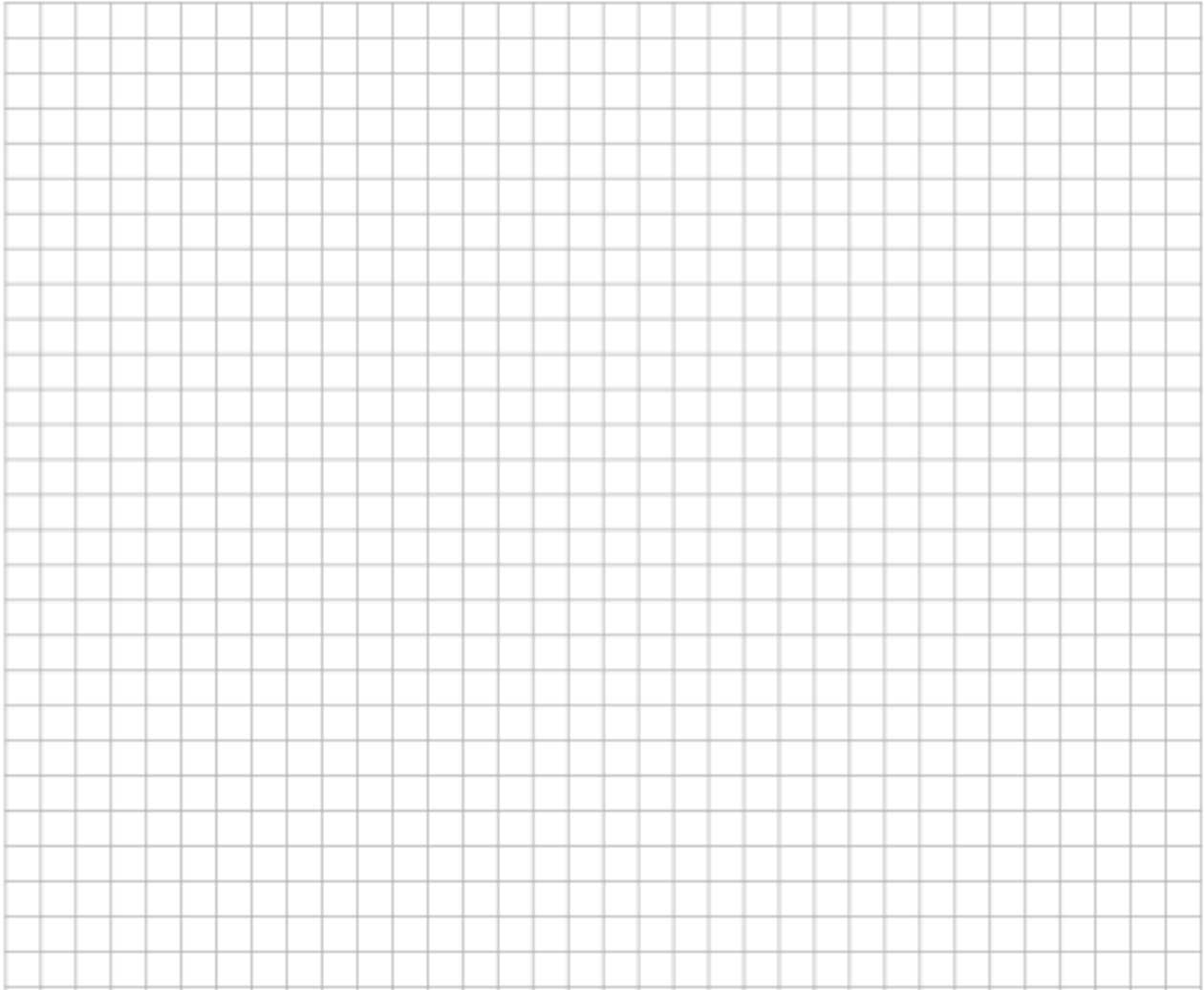
Transformations

1. Translations
2. Central symmetry
3. Axial symmetry

Translations

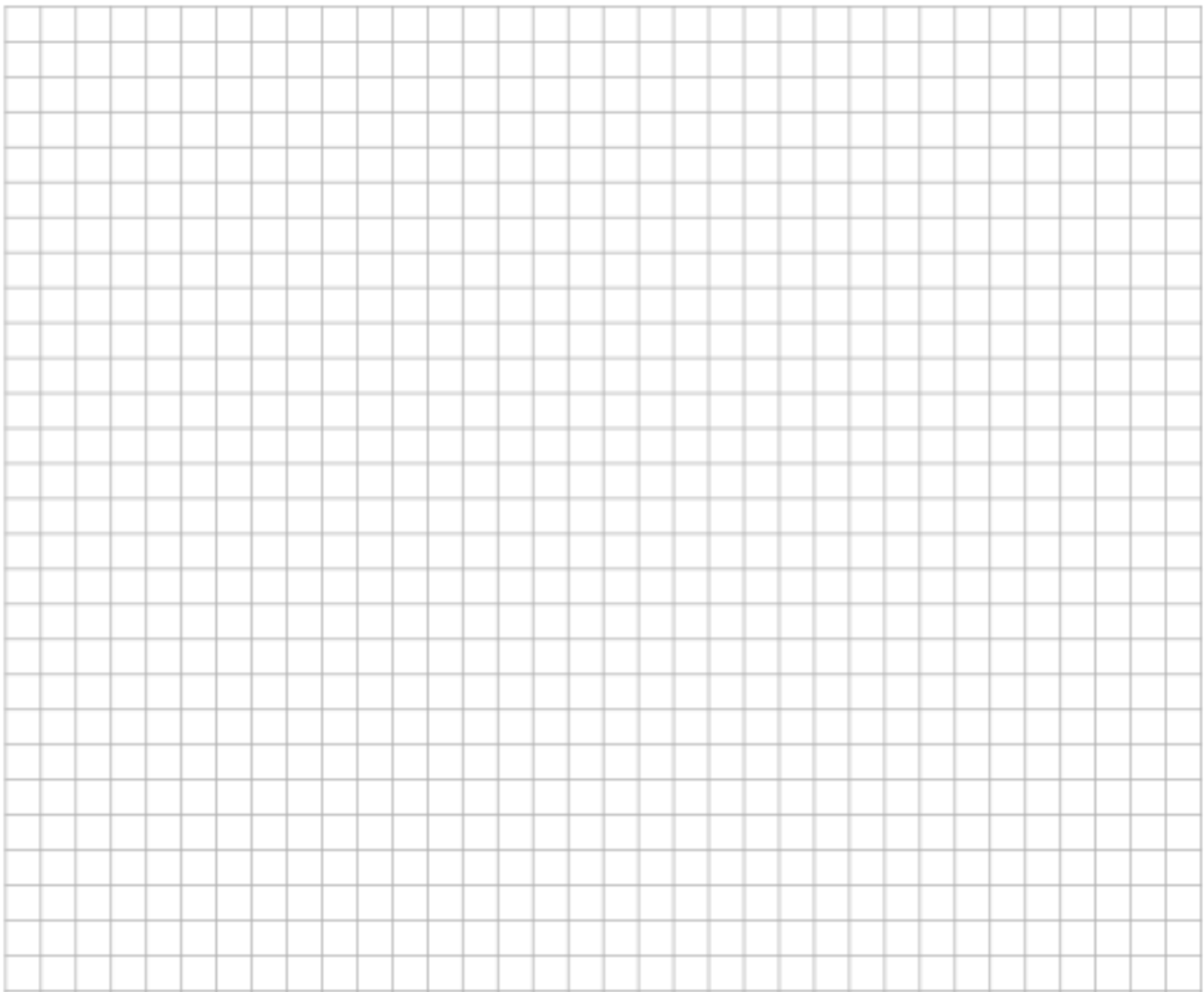
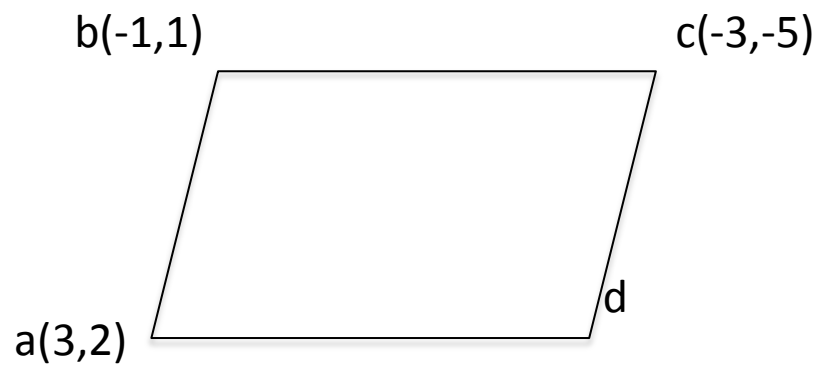
A translation means moving a point in a straight line.

1. $a(2,5)$ and $b(4,7)$ are two points. Find the image of the point $(5,2)$ under the translation $\begin{matrix} \rightarrow \\ ab \end{matrix}$



We can use translations to find a missing coordinant in a parallelogram.

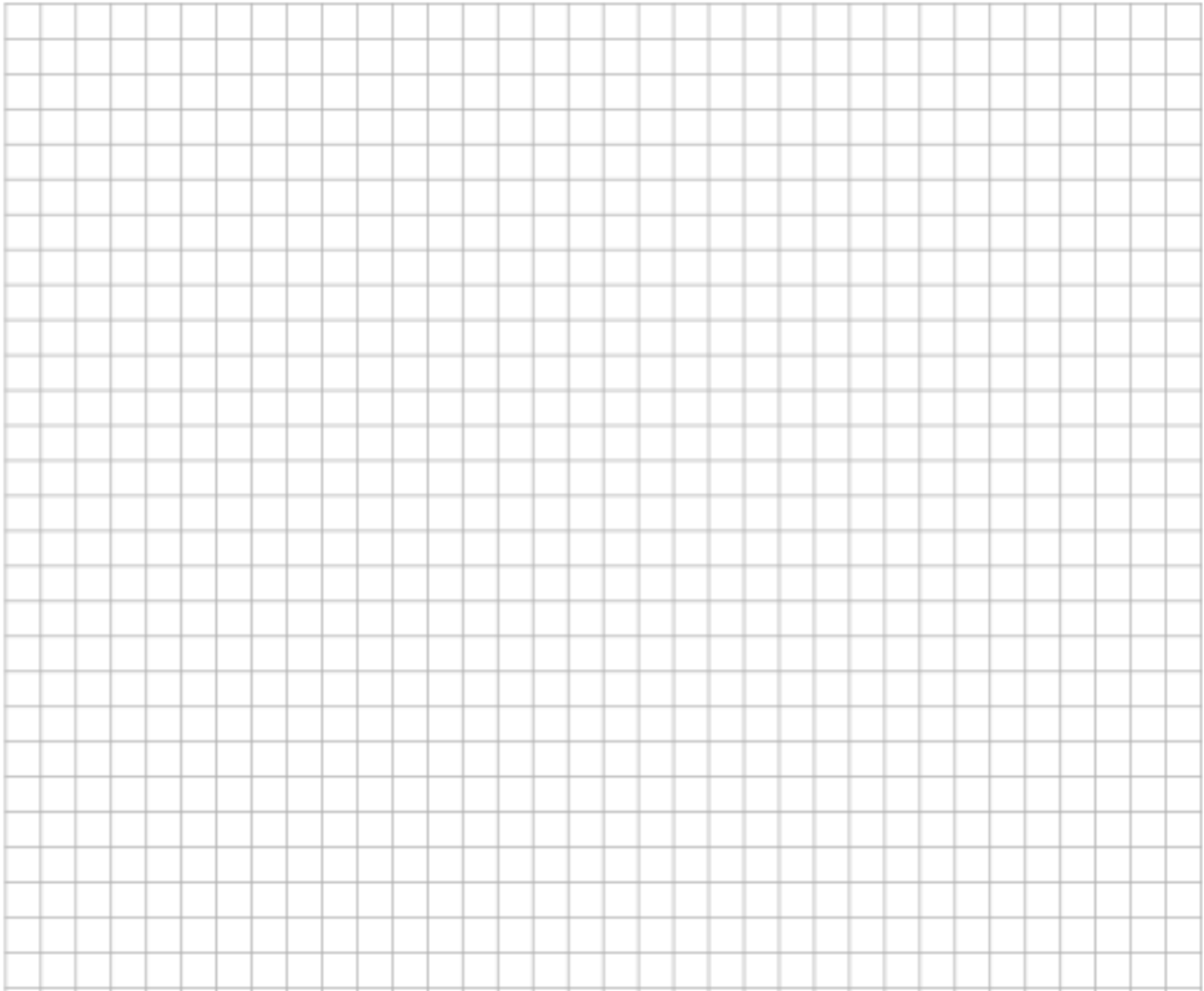
2. $a(3,2)$, $b(-1,1)$ and $c(-3,-5)$ are three vertices in a parallelogram $abcd$. Find the coordinates of the point d



Central symmetry

Central symmetry is like a mirror that is a point.

1. Find the image of the point $p(2,5)$ under central symmetry in the point $q(-2,-1)$.



Axial symmetry

Axial symmetry is like a mirror that is a line.

There are 3 types of symmetry here:

1. Axial symmetry in the x-axis, S_x

When we are asked to do this for a point we change the sign in front of the y part of the point.

2. Axial symmetry in the y-axis, S_y

When we are asked to do this we change the sign in front of the x part of the point.

3. This method can also be used for central symmetry in the origin, S_o

When are asked to do this we change the signs in front of both parts of the point.

1. Find the image of (4,3) under: (i) S_x (ii) S_y (iii) S_o



The area of a triangle

$$\text{area of triangle} = \frac{1}{2} |x_1y_2 - x_2y_1|$$

To use this formula we must move one vertex of the triangle to (0, 0) and translate the other two points.

1. Find the area of the triangle created by the points a(1,-2), b(3,5) and c(-2,2).

