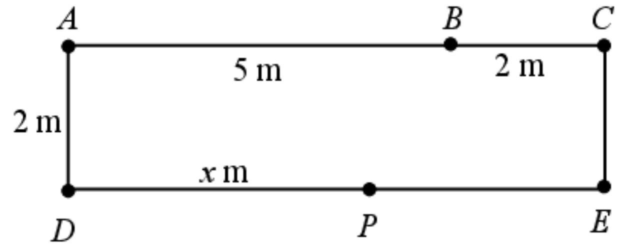
Question 1

(b) $ADEC$ is a rectangle with $|AC| = 7$ m and $|AD| = 2$ m, as shown.
B is a point on $[AC]$ such that $|AB| = 5$ m.
P is a point on $[DE]$ such that $|DP| = x$ m.



(i) Let $f(x) = |PA|^2 + |PB|^2 + |PC|^2$.

Show that $f(x) = 3x^2 - 24x + 86$, for $0 \le x \le 7$, $x \in \mathbb{R}$.

(ii) The function $f(x)$ has a minimum value at $x = k$.

Find the value of k and the minimum value of $f(x)$.

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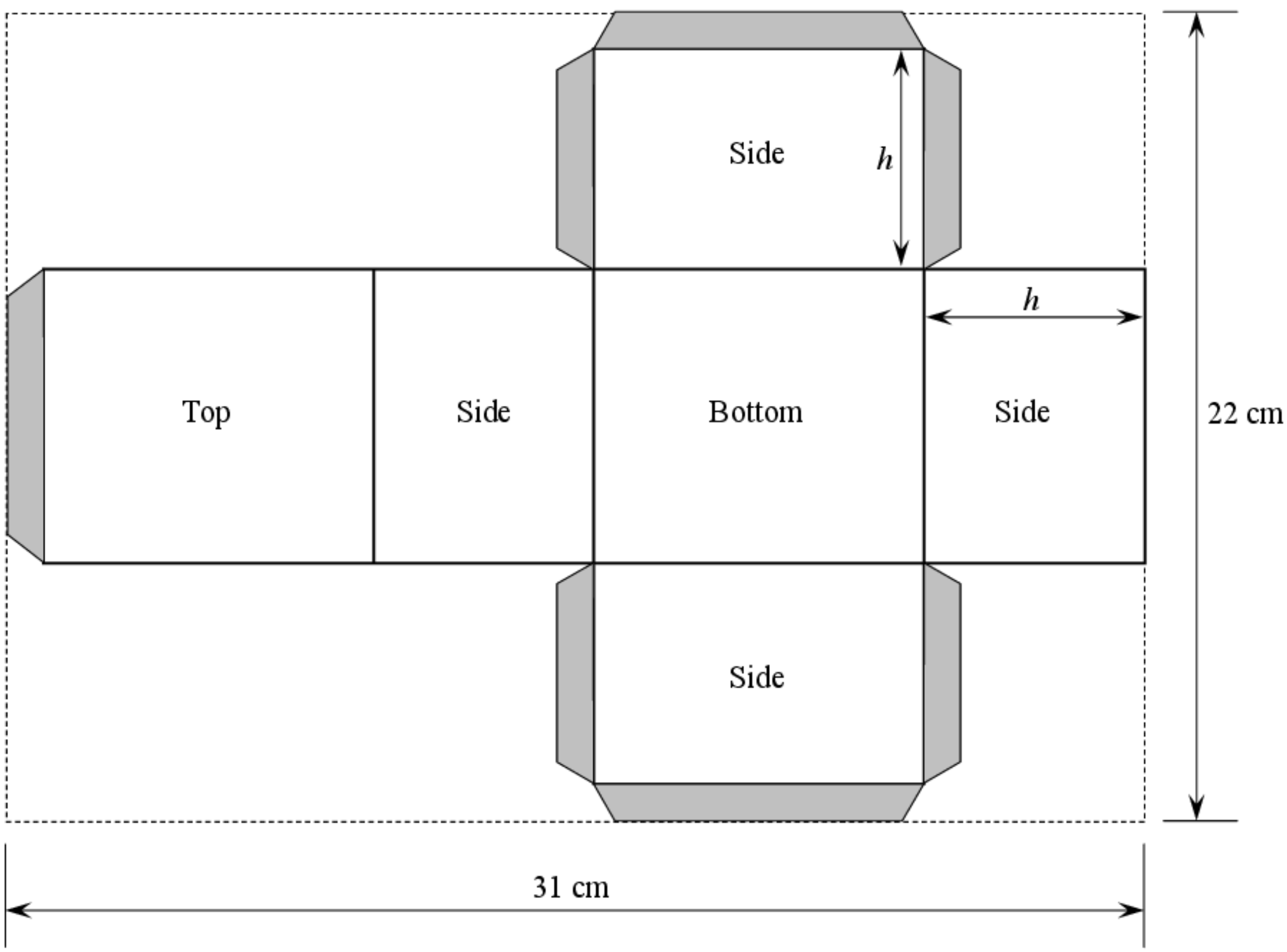
Question 7 **(50 marks)**

A company has to design a rectangular box for a new range of jellybeans. The box is to be assembled from a single piece of cardboard, cut from a rectangular sheet measuring 31 cm by 22 cm. The box is to have a capacity (volume) of 500 cm^3.

The net for the box is shown below. The company is going to use the full length and width of the rectangular piece of cardboard. The shaded areas are flaps of width 1 cm which are needed for assembly. The height of the box is h cm, as shown on the diagram.



(a) Write the dimensions of the box, in centimetres, in terms of h.

height = h cm

length = ______________ cm

width = ______________ cm

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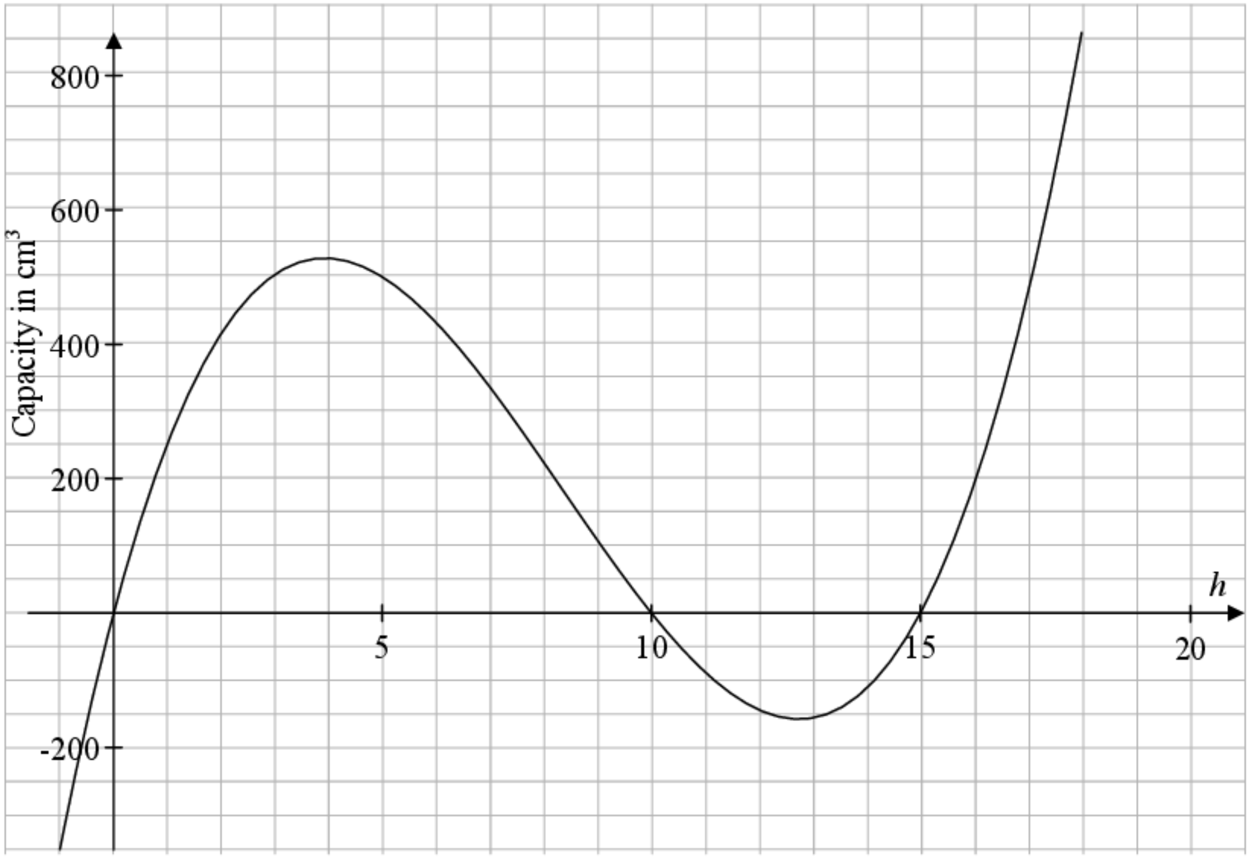
(b) Write an expression for the capacity of the box in cubic centimetres, in terms of h.

(c) Show that the value of h that gives a box with a square bottom will give the correct capacity.

(d) Find, correct to one decimal place, the other value of h that gives a box of the correct capacity.

(e) The client is planning a special "10% extra free" promotion and needs to increase the capacity of the box by 10%. The company is checking whether they can make this new box from a piece of cardboard the same size as the original one (31 cm × 22 cm). They draw the graph below to represent the box's capacity as a function of h. Use the graph to explain why it is *not* possible to make the larger box from such a piece of cardboard.



Explanation:

Question 3

Question 9 **(50 marks)**

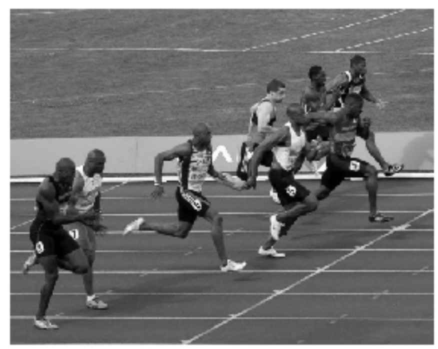
(a) Let $f(x) = -0{\cdot}5x^2 + 5x - 0{\cdot}98$, where $x \in \mathbb{R}$.

(i) Find the value of $f(0{\cdot}2)$.

(ii) Show that f has a local maximum point at $(5, 11{\cdot}52)$.

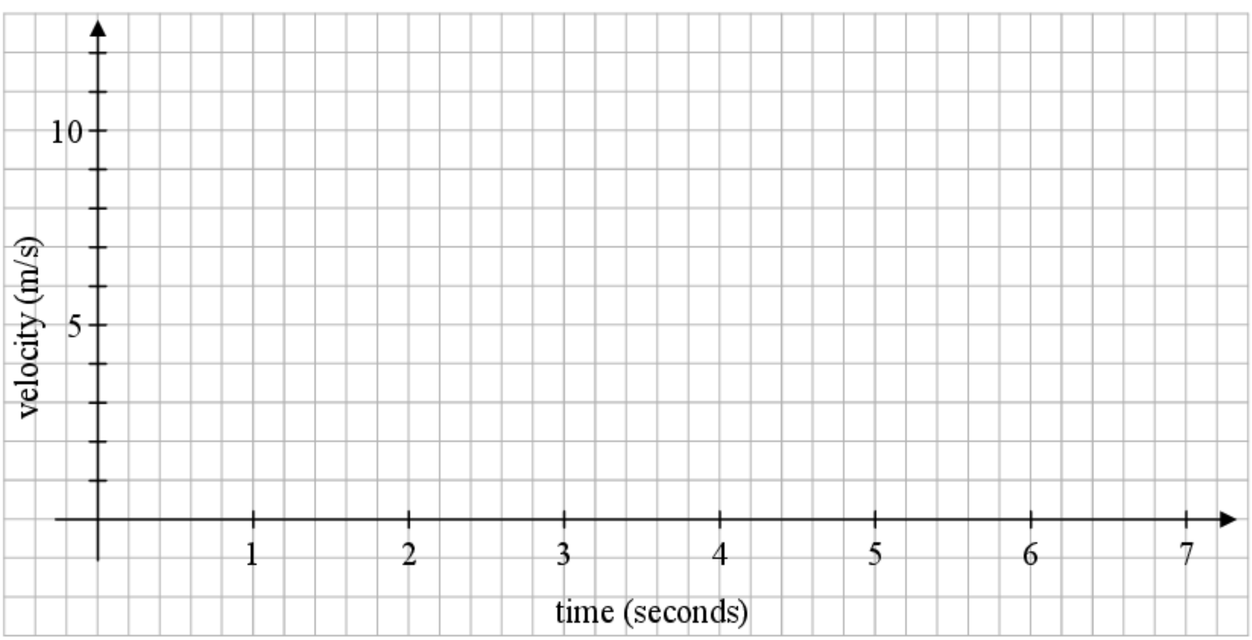
(b) A sprinter's velocity over the course of a particular 100 metre race is approximated by the following model, where v is the velocity in metres per second, and t is the time in seconds from the starting signal:

$$v(t) = \begin{cases} 0, & \text{for } 0 \le t < 0{\cdot}2 \\ -0{\cdot}5t^2 + 5t - 0{\cdot}98, & \text{for } 0{\cdot}2 \le t < 5 \\ 11{\cdot}52, & \text{for } t \ge 5 \end{cases}$$



Note that the function in part **(a)** is relevant to $v(t)$ above.

(i) Sketch the graph of v as a function of t for the first 7 seconds of the race.



(ii) Find the distance travelled by the sprinter in the first 5 seconds of the race.

(iii) Find the sprinter's finishing time for the race. Give your answer correct to two decimal places.

(c) A spherical snowball is melting at a rate proportional to its surface area. That is, the rate at which its volume is decreasing at any instant is proportional to its surface area at that instant.

(i) Prove that the radius of the snowball is decreasing at a constant rate.

(ii) If the snowball loses half of its volume in an hour, how long more will it take for it to melt completely?

Give your answer correct to the nearest minute.

Question 4

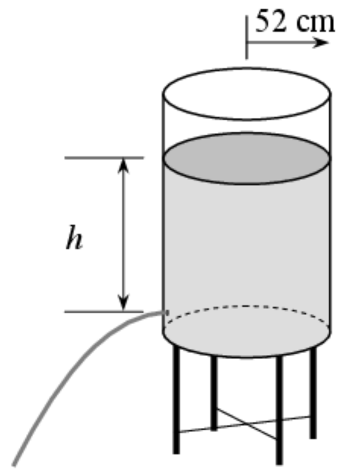
Question 7 **(50 marks)**

An open cylindrical tank of water has a hole near the bottom. The radius of the tank is 52 cm. The hole is a circle of radius 1 cm. The water level gradually drops as water escapes through the hole.

Over a certain 20-minute period, the height of the surface of the water is given by the formula

$$h = \left(10 - \frac{t}{200}\right)^2$$

where h is the height of the surface of the water, in cm, as measured from the centre of the hole,
and t is the time in seconds from a particular instant $t = 0$.



(a) What is the height of the surface at time $t = 0$?

(b) After how many seconds will the height of the surface be 64 cm?

(c) Find the rate at which the **volume** of water in the tank is decreasing at the instant when the height is 64 cm.
Give your answer correct to the nearest cm^3 per second.

(d) The rate at which the volume of water in the tank is decreasing is equal to the speed of the water coming out of the hole, multiplied by the area of the hole. Find the speed at which the water is coming out of the hole at the instant when the height is 64 cm.

(e) Show that, as t varies, the speed of the water coming out of the hole is a constant multiple of $\sqrt{h}$.

(f) The speed, in centimetres per second, of water coming out of a hole like this is known to be given by the formula

$$v = c\sqrt{1962h}$$

where c is a constant that depends on certain features of the hole.
Find, correct to one decimal place, the value of c for this hole.

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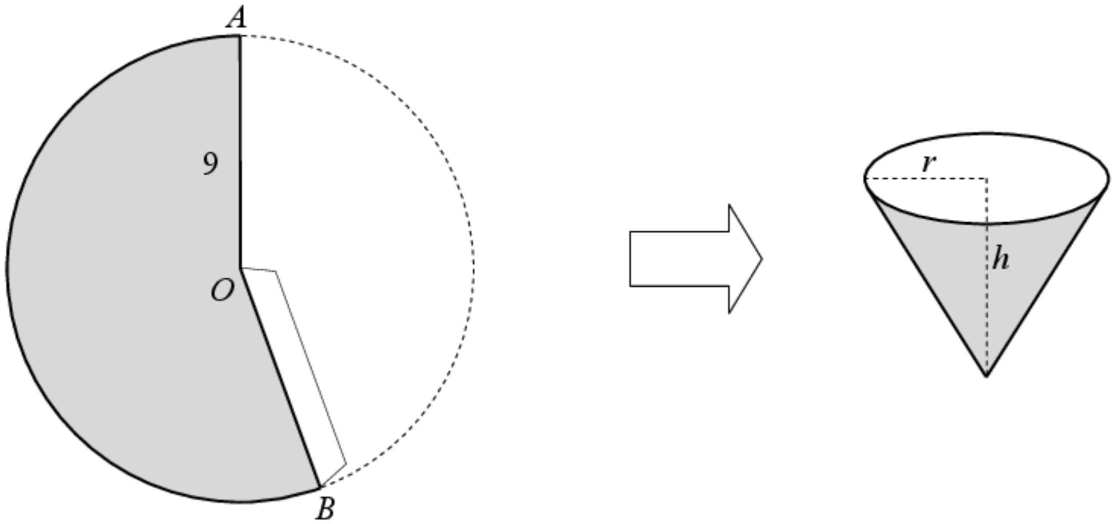
Question 5

Question 8 **(50 marks)**

A company uses waterproof paper to make disposable conical drinking cups. To make each cup, a sector AOB is cut from a circular piece of paper of radius 9 cm. The edges AO and OB are then joined to form the cup, as shown.

The radius of the rim of the cup is r, and the height of the cup is h.



(a) By expressing r^2 in terms of h, show that the capacity of the cup, in cm^3, is given by the formula

$$V = \frac{\pi}{3}h\left(81 - h^2\right).$$

(b) There are two positive values of h for which the capacity of the cup is $\frac{154\pi}{3}$.

One of these values is an integer.
Find the two values.
Give the non-integer value correct to two decimal places.

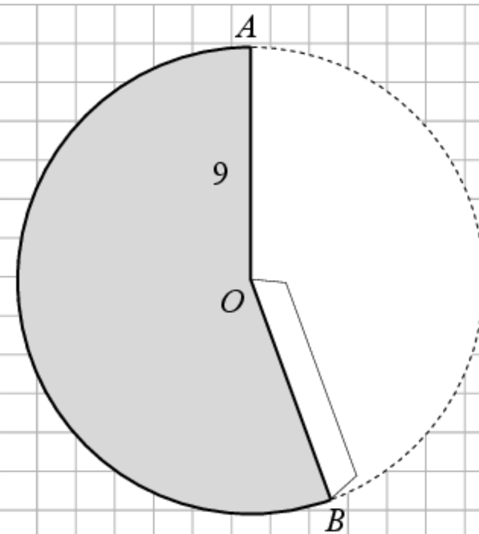
(c) Find the maximum possible volume of the cup, correct to the nearest cm^3.

(d) Complete the table below to show the radius, height, and capacity of each of the cups involved in parts **(b)** and **(c)** above.
In each case, give the radius and height correct to two decimal places.

	cups in part **(b)**		cup in part **(c)**
radius (r)			
height (h)			
capacity (V)	$\frac{154\pi}{3} \approx 161$ cm^3	$\frac{154\pi}{3} \approx 161$ cm^3	

(e) In practice, which one of the three cups above is the most reasonable shape for a conical cup? Give a reason for your answer.

(f) For the cup you have chosen in part **(e)**, find the measure of the angle *AOB* that must be cut from the circular disc in order to make the cup.
Give your answer in degrees, correct to the nearest degree.



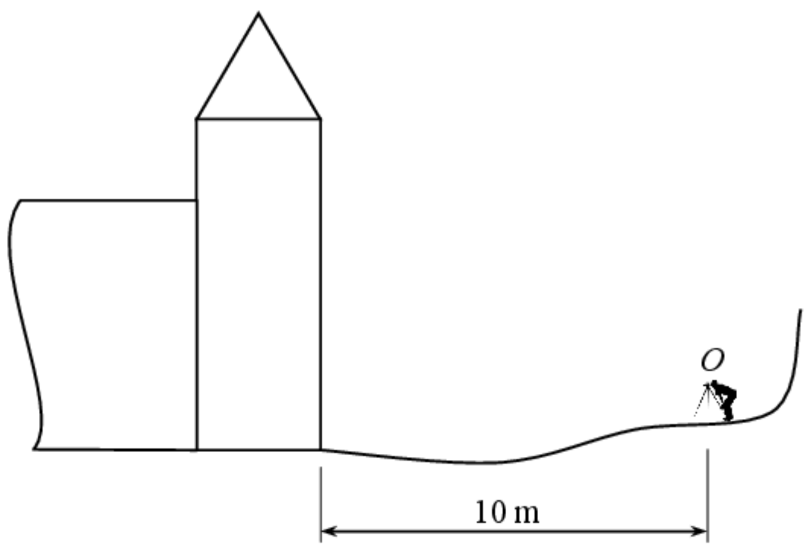
Question 6

Question 8 **(75 marks)**

(a) A tower that is part of a hotel has a square base of side 4 metres and a roof in the form of a pyramid. The owners plan to cover the roof with copper. To find the amount of copper needed, they need to know the total area of the roof.

A surveyor stands 10 metres from the tower, measured horizontally, and makes observations of angles of elevation from the point O as follows:

The angle of elevation of the top of the roof is 46°.
The angle of elevation of the closest point at the bottom of the roof is 42°.
The angle of depression of the closest point at the bottom of the tower is 9°.



(i) Find the vertical height of the roof.

(ii) Find the total area of the roof.

(iii) If all of the angles observed are subject to a possible error of ±1°, find the range of possible areas for the roof.

Question 7

Question 9 **(25 marks)**

A regular tetrahedron has four faces, each of which is an equilateral triangle.

A wooden puzzle consists of several pieces that can be assembled to make a regular tetrahedron. The manufacturer wants to package the assembled tetrahedron in a clear cylindrical container, with one face flat against the bottom.

If the length of one edge of the tetrahedron is $2a$, show that the volume of the smallest possible cylindrical container is $\left(\frac{8\sqrt{6}}{9}\right)\pi a^3$.