Algebra
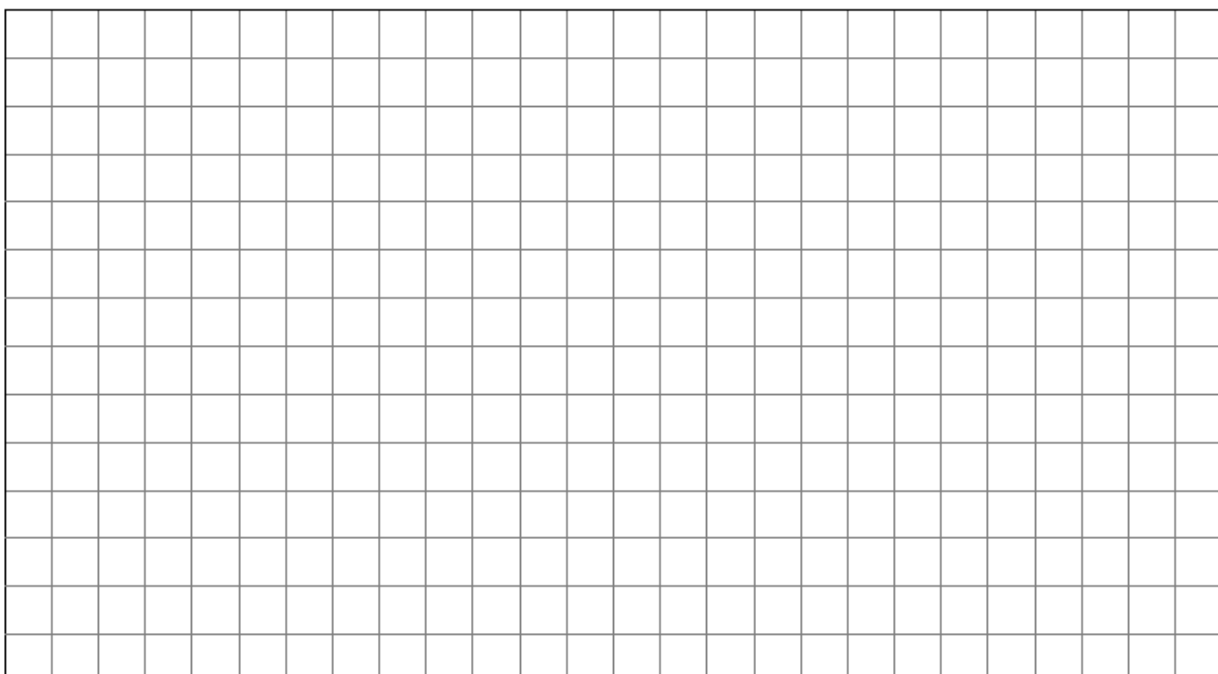
Name: _____

Question 1

Express $\dfrac{1-\sqrt{3}}{1+\sqrt{3}}$ in the form $a\sqrt{3}-b$, where a and $b \in \mathbf{N}$.

Question 2

Let $f(x) = x^3 + kx^2 - 4x - 12$, where k is a constant.

Given that $x+3$ is a factor of $f(x)$, find the value of k.

Question 3

Show that $p^3 + q^3 - (p+q)^3 = -3pq(p+q)$.

Question 4

Solve, without using a calculator, the following simultaneous equations:
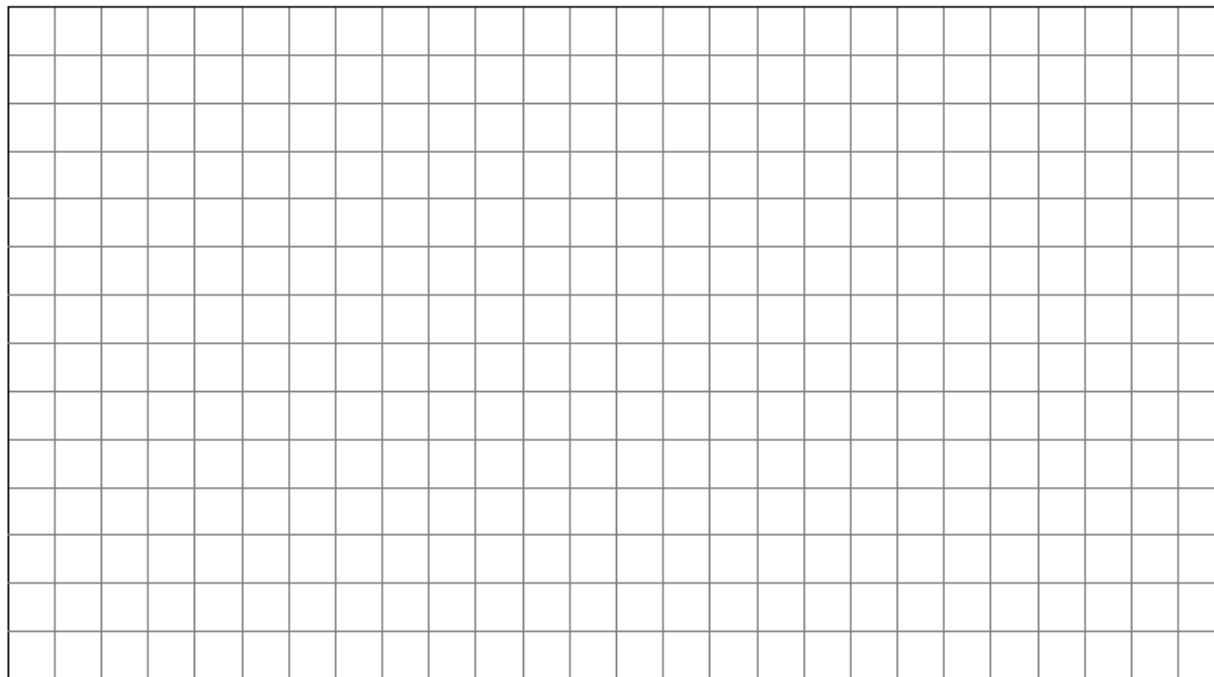
$$3x + y + z = 0$$
$$x - y + z = 2$$
$$2x - 3y - z = 9.$$

Question 5

Solve the simultaneous equations:

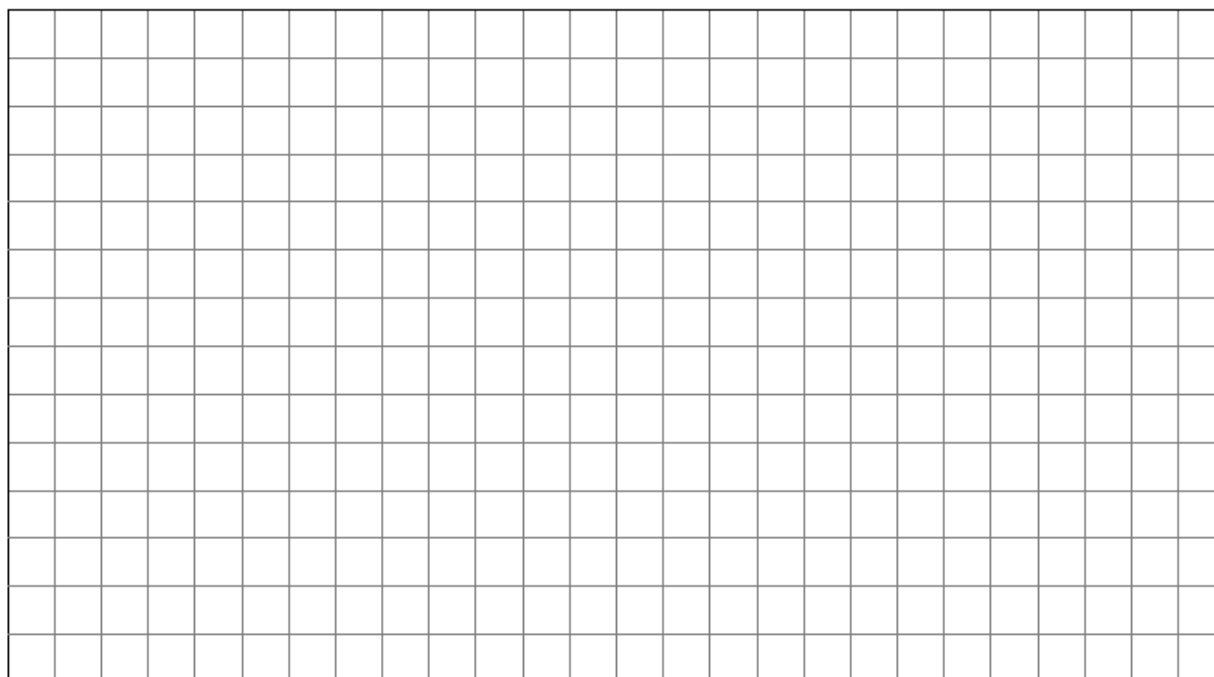$$\frac{x}{5} - \frac{y}{4} = 0$$

$$3x + \frac{y}{2} = 17.$$

Question 6

Find the real number a such that for all $x \neq 9$,

$$\frac{x-9}{\sqrt{x}-3} = \sqrt{x} + a.$$

Question 7

$f(x) = 3x^3 + mx^2 - 17x + n$, where m and n are constants.
Given that $x - 3$ and $x + 2$ are factors of $f(x)$, find the value of m and the value of n.

Question 8

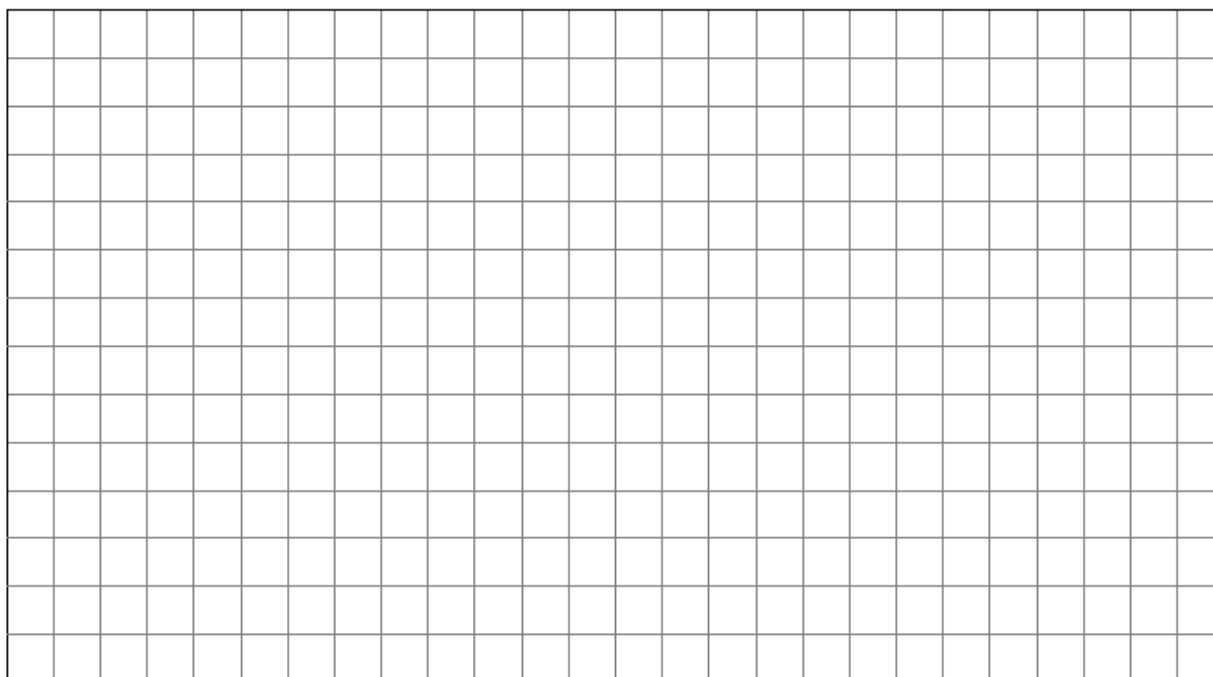Solve the simultaneous equations
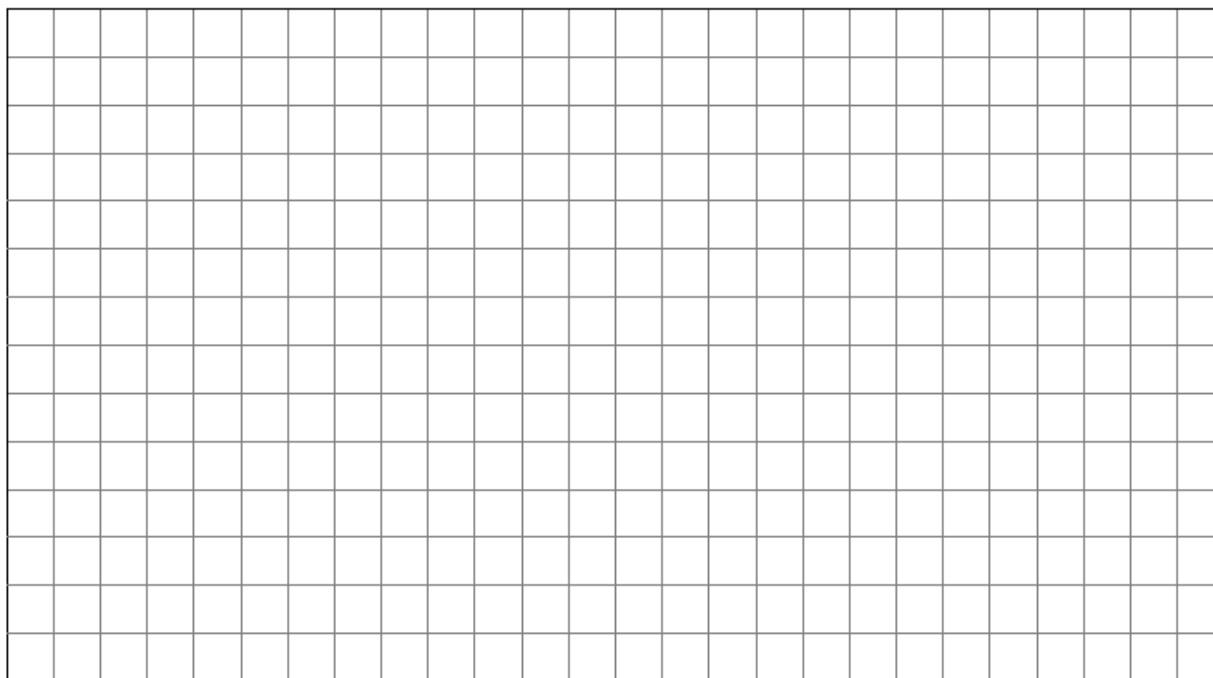
$$y = 2x - 5$$
$$x^2 + xy = 2.$$

Question 9

$x + p$ is a factor of both $ax^2 + b$ and $ax^2 + bx - ac$.

(i) Show that $p^2 = -\dfrac{b}{a}$ and that $p = \dfrac{-b - ac}{b}$.

(ii) Hence show that $p^2 + p^3 = c$.

Question 10

Solve the simultaneous equations

$$x + y + z = 2$$
$$2x + y + z = 3$$
$$x - 2y + 2z = 15.$$